Fuzzy Logic

Artificial Intelligence for Interactive Media and Games

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[Based on Buckland, Chapter 10]
Outline

- Background and Motivation
  - vagueness and discretization
  - application to weapon selection in Raven
  - fuzzy versus classical logic
  - DOM versus probabilities

- Theory and Algorithms
  - fuzzy set membership
  - linguistic variables
  - fuzzification and defuzzification
  - rule inference
Motivation

- Linguistic vagueness
  - “if the ball is far from the hole and the green is sloping gently downward from the left to the right, then hit the ball firmly and at an angle slightly to the left of the flag”

- Numerical discretization
  - Dumb: IQ < 90
  - Average: 90 ≤ IQ ≤ 110
  - Clever: 110 < IQ

  Should you call a person with IQ 89 dumb, but with 90 average ?!
Motivation

- Examples in Raven weapon selection
  
  * if the target is far and you have lots of ammo, then the rocket launcher is a desirable choice
  
  * if target is at medium range and you have lots of ammo, then the rocket launcher is a very desirable choice
Classic ("Crisp") Sets

- Universe (of discourse)
- Characteristic (membership) function (predicate)
  - even: $U \rightarrow \text{boolean}$
  - odd: $U \rightarrow \text{boolean}$
    - even(2) = T, even(3) = F, etc.
    - singleDigit(2) = T, singleDigit(10) = F, etc.
- Operators: union, intersection, complement
  - and, or, not for characteristic predicates
Fuzzy Set Membership

- Range of membership function for each set generalized from boolean to real interval \([0,1]\)
  - dumb: IQ \(\Rightarrow [0, 1]\)
  - average: IQ \(\Rightarrow [0, 1]\)
  - clever: IQ \(\Rightarrow [0, 1]\)

- A given value can be a member of more than one set with different degrees, e.g.,
  - \(\text{dumb}(20) = 1.0\) \hspace{1cm} \(\text{average}(20) = 0.0\) \hspace{1cm} \(\text{clever}(20) = 0.0\)
  - \(\text{dumb}(89) = 0.5\) \hspace{1cm} \(\text{average}(89) = 0.5\) \hspace{1cm} \(\text{clever}(89) = 0.0\)
  - \(\text{dumb}(90) = 0.5\) \hspace{1cm} \(\text{average}(89) = 0.5\) \hspace{1cm} \(\text{clever}(90) = 0.0\)

- degree of membership (DOM) shifts gradually as value changes
Fuzzy Set Membership

- membership functions can be any shape
- but for given value, degrees of membership in all sets (in FLV, tbd) should sum to 1.0
what would fuzzy set membership diagram look like for “crisp” version of IQ?
Membership Function Shapes

- Triangular
- Trapezoidal
- S-Curve
- Left Shoulder
- Right Shoulder
- Singleton
Membership versus Probability

- Fuzzy logic closely related to probabilistic logics
  - both use real interval [0, 1]
- Probability reflects *uncertainty of outcome*
  - especially for repeated events (coin toss, etc.)
  - if I reach in a bag with 1 green ball and 3 red balls, there is a 25% chance I will pull out a green ball
- Fuzzy sets reflect *conceptual uncertainty*
  - e.g., is this weird-colored ball green or red?
  - “confidence value”
Fuzzy Set Operators

\[ F_{A \cap B}(x) = \min\{F_A(x), F_B(x)\} \]
Fuzzy Set Operators

\[ F_{A \cup B} (x) = \max \{ F_A (x), F_B (x) \} \]
Fuzzy Set Operators

\[ F^{-}_A(x) = 1 - F^+_A(x) \]
Hedges

\[ F_{\text{Very}(A)}(x) = [F_A(x)]^2 \]

\[ F_{\text{Fairly}(A)}(x) = \sqrt{F_A(x)} \]
Fuzzy Linguistic Variable

- conceptual grouping of several fuzzy sets (membership functions) with the same domain (universe)
  - IQ = { dumb, average, clever }
Target Heading Variable

![Diagram showing target heading variable with degrees ranging from -15 to 15 degrees, with labels for Far Left, Left, Center, Right, and Far Right.]
Linguistic Variable Design Guidelines

BAD: values don’t add to 1.0

BAD: values belong to more than two sets
Fuzzy Rules

IF *antecedent* THEN *consequent*

- degree of *membership* of given value in the *antecedent* set determines the degree of *confidence* in the *consequent*

- *antecedent* and consequent may be primitive fuzzy sets or expressions composed with operators
Fuzzy Rules

IF Target_isFarRight THEN Turn_QuicklyToRight

IF Very(Enemy_BadlyInjured) THEN Behavior_Aggressive

IF Ball_isCloseToHole AND Green_isLevel THEN
    HitBall_Gently AND HitBall_DirectlyAtHole

IF Target_Medium AND Ammo_Low THEN
    RocketLauncher_Desirable
Fuzzy Rule Inference

1. **Fuzzification**
2. **Fuzzy Rules**
3. **Defuzzification**

**Crisp** input flows through these stages to produce a **crisp** output.
Raven Weapon Selection Example

1. Decide on antecedent and consequent linguistic variables

2. Design fuzzy membership functions for each variable

3. Define rules using variables
Raven Weapon Selection Example

- Weapon selection depends on (antecedents):
  - distance to target
  - ammo status

- Conclusion (consequent) is
  - desirability of weapon

- Some FLV’s shared for all weapons:
  - distance to target
  - desirability of weapon

- Some FLV’s per weapon:
  - ammo status

- Separate sets of rules for each weapon
Designing Membership Functions

Distance to Target

Close  Medium  Far

0  100  200  300  400  500

0  1
Designing Membership Functions

(for Rocket Launcher)
Designing Membership Functions

![Membership Functions Diagram]

- Undesirable
- Desirable
- Very Desirable

Desirability
Fuzzy Rule Inference
Rocket Launcher Selection Rules

(1) IF Target_Far AND Ammo_Loads THEN Desirable
(2) IF Target_Far AND Ammo_Okay THEN Undesirable
(3) IF Target_Far AND Ammo_Low THEN Undesirable
(4) IF Target_Medium AND Ammo_Loads THEN VeryDesirable
(5) IF Target_Medium AND Ammo_Okay THEN VeryDesirable
(6) IF Target_Medium AND Ammo_Low THEN Desirable
(7) IF Target_Close AND Ammo_Loads THEN Undesirable
(8) IF Target_Close AND Ammo_Okay THEN Undesirable
(9) IF Target_Close AND Ammo_Low THEN Undesirable

* can reduce to 6 rules by Comb’s Method
1. **Given an input value for each linguistic variable used in the rule antecedents**

2. **For each rule**
   - calculate degree of confidence in conclusion from degree of membership of input value in antecedent

3. **Combine** all the inferred conclusions into a single fuzzy variable

4. **Defuzzify** the conclusion to single (crisp) output value
Running the Rules

- Input values:
  - Distance to Target = 200 pixels
  - Ammo = 8 rockets

- Rule (1)
  IF Target_Far AND Ammo_Loads THEN Desirable

  - Target_Far(200) = 0.33
  - Ammo_Loads(8) = 0.0
  - Desirable = Target_Far(200) AND Ammo_Loads(8)
    = MIN(0.33, 0.0) = 0.0
Running the Rules

- **Rule (1)**
  IF Target_Far AND Ammo_Loads THEN Desirable
  - Target_Far(200) = 0.33
  - Ammo_Loads(8) = 0.0
  - Desirable = MIN(0.33, 0.0) = 0.0
Running the Rules

- Rule (2)
  IF Target_Far AND Ammo_Okay THEN Undesirable
Running the Rules

- Rule (3)
  IF Target_Far AND Ammo_Low THEN Undesirable
### Fuzzy Associative Matrix

**Target_Close** | **Target_Medium** | **Target_Far**
---|---|---
Undesirable | Desirable | Undesirable
0 | 0.2 | 0.2

Undesirable | VeryDesirable | Undesirable
0 | 0.67 | 0.33

Undesirable | VeryDesirable | Desirable
0 | 0 | 0

- Shaded cells from “fired” (non-zero) rules
- What should confidence level be for Undesirable?
- How to combine output values?
- Use MAX(0.2,0.33)
Inferred Consequent Sets

- results of each rule OR’ed together with underlying membership function
- “clips” each shape
Combined Output Variable

- output variable could feed into other rules
- or if the end of the line, extract a single value ("defuzz")
Defuzzification – Extracting a single value

- **Average of Maxima (MaxAv)**
  - good approximation to centroid
  - weighted sum of representative values

\[
\sum \left( \text{representativeValue} \times \text{confidence} \right) \div \sum \text{confidence}
\]

Rocket Launcher
Desirability = 60.625
Distance to target = 200 pixels
Ammo = 8 rockets

Rocket Launcher Desirability = 60.625
Implementation Classes

- FuzzyModule
- FuzzySet
- FuzzyVariable
- FuzzyTerm
- Fuzzy Operator
- Fuzzy Rule

...for weapon selection in Raven
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<th>Topic</th>
<th>Details</th>
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<tr>
<td>Mon, Feb 29</td>
<td>Chapter 10</td>
<td>Fuzzy Logic</td>
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<tr>
<td>Tue, Mar 1</td>
<td>Special Guest: Damian Isla</td>
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<tr>
<td>Weds, Mar 2</td>
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<td>(Due 6pm!) 12 - Tournament Bot [10%]</td>
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<td>Thu, Mar 3</td>
<td>Raven Tournament (GH 012)</td>
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<td>Fri, Mar 4</td>
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