CS3133
Homework #4

I worked with:

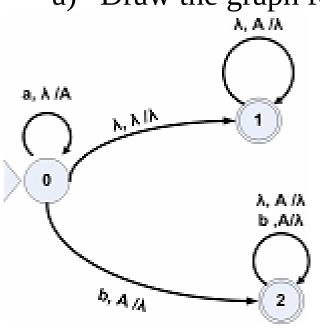
I consulted:

#1. a) Given the following PDA, M:

$Q = \{q_0, q_1, q_2\}$
$\Sigma = \{a,b\}$
$\Gamma = \{A\}$
$F = \{q_1, q_2\}$

$\delta(q_0,a,\lambda) = \{[q_0,A]\}$
$\delta(q_0, \lambda,\lambda) = \{[q_1, \lambda]\}$
$\delta(q_0,b,A) = \{[q_2, \lambda]\}$
$\delta(q_1, \lambda,A) = \{[q_1, \lambda]\}$
$\delta(q_2, b,A) = \{[q_2, \lambda]\}$
$\delta(q_2, \lambda,A) = \{[q_2, \lambda]\}$

a) Draw the graph for M



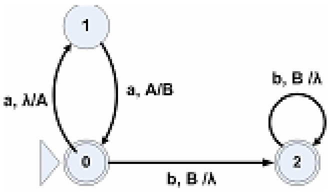
b) Trace the computations of *aab, abb, aba, aabb*

[q0,aab,λ] → [q0,ab,A] → [q0,b,A] → [q2, λ, λ] (accepted)
[q0,abb,λ] → [q0,bb,A] → [q0,b, λ] → [q1, b , λ] (rejected)
[q0,aba,λ] → [q0,ba,A] → [q0,a, λ] → [q1, a , λ] (rejected)
[q0,aabb,λ] → [q0,abb,A] → [q0,bb,AA] → [q2,b,A] → [q2, λ, λ] (accepted)

b) What is L(M)?

$L(M) = \{a^n b^m \mid n \geq m \geq 0\}$

#2. a) Construct a PDA to accept $\{a^{2i}b^{i} \mid i \geq 0\}$



b) Show computations on *a a b* and *a b b*

[q0,aab,λ] → [q1,ab,A] → [q0,b,B] → [q0, λ, λ] (accepts)

[q0,abb,λ] → → [q1,bb,A] → halts (rejects)

#3. Show context free languages are closed under reversal.

If L is a CFL, there is a grammar, G, with L = L(G).
For any production, A → α in G, create a new grammar with
$A \rightarrow \alpha^R$

For $L = \{a^n b^n \mid n \geq 0\}$, G is

S → a S b | λ

and G for $L^R = \{b^n a^n \mid n \geq 0\}$, is

S → b S a | λ

#4. #4. Use the pumping lemma to show that $L = \{w w^R w \mid w \in \{a,b\}^* \}$ is not context-free.

If L were regular, then there is a constant, k, such that if z ε L and $|z| \geq k$,
the PL conditions are true.
Pick $z = a^n b^n b^n a^n a^n b^n$. Then z ε L and $|z| \geq k$
So
z = *u v w x y* with

$|v\,w\,x| \leq k$
$|v| + |y| > 0$ (i.e., not both v and x are λ)

There are two cases:

Case 1 *v w x* is completely within one of the a^n or b^n's, say the first b^n

Assume |v| > 0 so v = b^p with p >0. (Results will be similar if |x| > 0)

Then *u v v w x x y* = $a^n b^{n+p} b^n a^n a^n b^n$ and there is no way to split this up to be of the form: w w^R w

Case 2 z overlaps a's and b's or b's and a's: then either first and last w will not be the same or again no way to split this up to be of the form: w w^R w

#5. **Given a transition function δ(q,a), defined on a symbol *a*:**

a) **(1 point) Define the extended transition function $\delta^*(q,w)$, defined on strings *w* (you may use either the text's definition or the one used in class)**

See class notes and/or text

b) **(7 points) Prove using induction that $\delta^*(q, w_1w_2) = \delta^*(\delta^*(q,w_1),w_2)$. State clearly what you are doing the induction on, set the proof up clearly and give reasons for each step.**

Done in class (Monday, Sept. 11)

c) **(2 points) Use part b and the fact that δ*(q,a) = δ(q,a) to show δ*(q,aw) = δ*(δ(q,a),w)**

δ*(q,aw) = δ*(δ*(q,a),w)   by part b
= δ*(δ(q,a),w)   given that δ*(q,a) = δ (q,a)