Top-Down Parsing

- recursive descent
- predictive parsing

Lecture 9

Lecture Outline

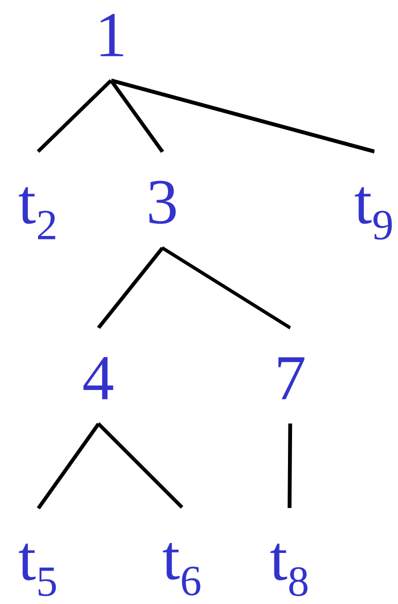
- Implementation of parsers
- Two approaches
  - Top-down
  - Bottom-up
- Today: Top-Down
  - Easier to understand and program manually
- Then: Bottom-Up
  - More powerful and used by most parser generators

Intro to Top-Down Parsing

- The parse tree is constructed
  - From the top
  - From left to right

- Terminals are seen in order of appearance in the token stream:

  $t_2 \;\; t_5 \;\; t_6 \;\; t_8 \;\; t_9$



Recursive Descent Parsing

- Consider the grammar

$$E \to T + E \mid T$$
$$T \to int \mid int * T \mid (E)$$

- Token stream is: $int_5 * int_2$
- Start with top-level non-terminal E

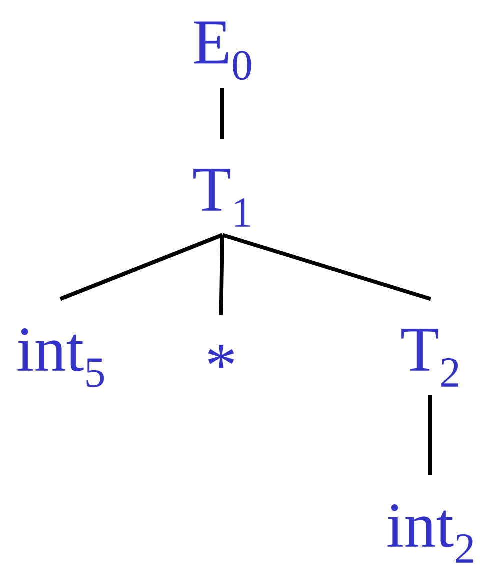
- Try the rules for E in order

Recursive Descent Parsing. Example (Cont.)

- Try $E_0 \rightarrow T_1 + E_2$
- Then try a rule for $T_1 \rightarrow (E_3)$
  - But (does not match input token int_5
- Try $T_1 \rightarrow$ int . Token matches.
  - But + after T_1 does not match input token *
- Try $T_1 \rightarrow$ int * T_2
  - This will match but + after T_1 will be unmatched
- Has exhausted the choices for T_1
  - Backtrack to choice for E_0

Recursive Descent Parsing. Example (Cont.)

- Try $E_0 \rightarrow T_1$
- Follow same steps as before for T_1
  - And succeed with $T_1 \rightarrow$ int * T_2 and $T_2 \rightarrow$ int
  - With the following parse tree



A Recursive Descent Parser. Preliminaries

- Let TOKEN be the type of tokens
  - Special tokens INT, OPEN, CLOSE, PLUS, TIMES

- Let the global next point to the next token

A Recursive Descent Parser (2)

- Define boolean functions that check the token string for a match of
  - A given token terminal

    ```
    bool term(TOKEN tok) { return *next++ == tok; }
    ```
  - A given production of S (the n^{th})

    bool S_n() { … }
  - Any production of S:

    ```
    bool S() { … }
    ```

- These functions advance next

A Recursive Descent Parser (3)

- For production $E \rightarrow T$

```
bool E1() { return T(); }
```

- For production $E \rightarrow T + E$

```
bool E2() { return T() && term(PLUS) && E(); }
```

- For all productions of E (with backtracking)

```
bool E() {
  TOKEN *save = next;
  return    (next = save, E1())
         || (next = save,  E2());   }
```

A Recursive Descent Parser (4)

- Functions for non-terminal T

```
bool T1() { return term(OPEN) && E() && term(CLOSE); }
bool T2() { return term(INT) && term(TIMES) && T(); }
bool T3() { return term(INT); }

  bool T() {
    TOKEN *save = next;
    return    (next = save, T1())
           || (next = save,  T2())
           || (next = save,  T3()); }
```

Recursive Descent Parsing. Notes.

- To start the parser
  - Initialize next to point to first token
  - Invoke E()
- Notice how this simulates our previous example

- Easy to implement by hand
- But does not always work ...

When Recursive Descent Does Not Work

- Consider a production $S \rightarrow S\ a$

```
bool S1() { return S() && term(a); }
bool S() { return  S1(); }
```

- S() will get into an infinite loop

- A left-recursive grammar has a non-terminal S

  $S \rightarrow^{+} S\alpha$ for some α

- Recursive descent does not work in such cases

Elimination of Left Recursion

- Consider the left-recursive grammar

$$S \rightarrow S\ \alpha \mid \beta$$

- S generates all strings starting with a β and followed by a number of α

- Can rewrite using right-recursion

$$S \rightarrow \beta\ S'$$

$$S' \rightarrow \alpha\ S' \mid \varepsilon$$

More Elimination of Left-Recursion

- In general

$$S \rightarrow S\ \alpha_1 \mid \dots \mid S\ \alpha_n \mid \beta_1 \mid \dots \mid \beta_m$$

- All strings derived from S start with one of $\beta_1,\dots,\beta_m$ and continue with several instances of $\alpha_1,\dots,\alpha_n$
- Rewrite as

$$S \rightarrow \beta_1\ S' \mid \dots \mid \beta_m\ S'$$

$$S' \rightarrow \alpha_1\ S' \mid \dots \mid \alpha_n\ S' \mid \varepsilon$$

General Left Recursion

- The grammar

$$S \to A\ \alpha \mid \delta$$

$$A \to S\ \beta$$

  is also left-recursive because

$$S \to^{+} S\ \beta\ \alpha$$

- This left-recursion can also be eliminated
- See book, Section 4.3 for general algorithm

Summary of Recursive Descent

- Simple and general parsing strategy
  - Left-recursion must be eliminated first
  - … but that can be done automatically
- Unpopular because of backtracking
  - Thought to be too inefficient

- In practice, backtracking is eliminated by restricting the grammar

Predictive Parsers

- Like recursive-descent but parser can "predict" which production to use
  - By looking at the next few tokens
  - No backtracking
- Predictive parsers accept LL(k) grammars
  - L means "left-to-right" scan of input
  - L means "leftmost derivation"
  - k means "predict based on k tokens of lookahead"
- In practice, LL(1) is used

LL(1) Languages

- In recursive-descent, for each non-terminal and input token there may be a choice of production
- LL(1) means that for each non-terminal and token there is only one production
- Can be specified via 2D tables
  - One dimension for current non-terminal to expand
  - One dimension for next token
  - A table entry contains one production

Predictive Parsing and Left Factoring

- Recall the grammar

  $E \rightarrow T + E \mid T$

  $T \rightarrow int \mid int * T \mid (E)$

- Hard to predict because
  - For T two productions start with int
  - For E it is not clear how to predict

- A grammar must be left-factored before use for predictive parsing

Left-Factoring Example

- Recall the grammar

$$E \rightarrow T + E \mid T$$
$$T \rightarrow int \mid int * T \mid (E)$$

- Factor out common prefixes of productions

$$E \rightarrow T\ X$$
$$X \rightarrow + E \mid \varepsilon$$
$$T \rightarrow (E) \mid int\ Y$$
$$Y \rightarrow * T \mid \varepsilon$$

LL(1) Parsing Table Example

- Left-factored grammar

  $E \to T\ X$ $\qquad$ $X \to +\ E \mid \varepsilon$

  $T \to (\ E\ ) \mid int\ Y$ $\qquad$ $Y \to *\ T \mid \varepsilon$

- The LL(1) parsing table:

	int	*	+	(	)	$
E	T X			T X		
X			+ E		ε	ε
T	int Y			(E)		
Y		* T	ε		ε	ε

LL(1) Parsing Table Example (Cont.)

- Consider the [E, int] entry
  - "When current non-terminal is E and next input is int, use production $E \rightarrow T X$
  - This production can generate an int in the first place
- Consider the [Y,+] entry
  - "When current non-terminal is Y and current token is +, get rid of Y"
  - Y can be followed by + only in a derivation in which $Y \rightarrow \varepsilon$

LL(1) Parsing Tables. Errors

- Blank entries indicate error situations
  - Consider the [E,*] entry
  - “There is no way to derive a string starting with * from non-terminal E”

Using Parsing Tables

- Method similar to recursive descent, except
  - For each non-terminal S
  - We look at the next token a
  - And chose the production shown at [S,a]
- We use a stack to keep track of pending non-terminals
- We reject when we encounter an error state
- We accept when we encounter end-of-input

LL(1) Parsing Algorithm

```
initialize stack = <S $> and next
repeat
   case stack of
      <X, rest>  : if T[X,*next] = Y1...Yn
                      then stack ← <Y1... Yn rest>;
                      else  error ();
      <t, rest>   : if t == *next ++
                      then  stack ← <rest>;
                      else error ();
until stack == < >
```

LL(1) Parsing Example

Stack	Input	Action
E $	int * int $	T X
T X $	int * int $	int Y
int Y X $	int * int $	terminal
Y X $	* int $	* T
* T X $	* int $	terminal
T X $	int $	int Y
int Y X $	int $	terminal
Y X $	$	ε
X $	$	ε
$	$	ACCEPT

Constructing Parsing Tables

- LL(1) languages are those defined by a parsing table for the LL(1) algorithm
- No table entry can be multiply defined

- We want to generate parsing tables from CFG

Constructing Parsing Tables (Cont.)

- If $A \to \alpha$, where in the line of A we place α ?
- In the column of t where t can start a string derived from α
  - $\alpha \to^{*} t\ \beta$
  - We say that $t \in First(\alpha)$
- In the column of t if α is ε and t can follow an A
  - $S \to^{*} \beta\ A\ t\ \delta$
  - We say $t \in Follow(A)$

Computing First Sets

Definition: $First(X) = \{ t \mid X \rightarrow^* t\alpha\} \cup \{\varepsilon \mid X \rightarrow^* \varepsilon\}$

Algorithm sketch (see book for details):

1. for all terminals t do $First(t) \leftarrow \{ t \}$
2. for each production $X \rightarrow \varepsilon$ do $First(X) \leftarrow \{ \varepsilon \}$
3. if $X \rightarrow A_1 \dots A_n\ \alpha$ and $\varepsilon \in First(A_i)$, $1 \le i \le n$ do
   - add $First(\alpha)$ to $First(X)$
4. for each $X \rightarrow A_1 \dots A_n$ s.t. $\varepsilon \in First(A_i)$, $1 \le i \le n$ do
   - add ε to $First(X)$
5. repeat steps 4 & 5 until no First set can be grown

First Sets. Example

- Recall the grammar

$E \to T\ X \qquad\qquad X \to +\ E \mid \varepsilon$

$T \to (\ E\ ) \mid \text{int}\ Y \qquad\qquad Y \to *\ T \mid \varepsilon$

- First sets

First(( ) = { (}   First(T) = {int, (}

First( )) = {) }   First(E) = {int, (}

First(int) = { int }   First(X) = {+, ε }

First(+) = { + }   First(Y) = {*, ε }

First(*) = { * }

Computing Follow Sets

- Definition:

  $$\text{Follow}(X) = \{\ t \mid S \rightarrow^{*} \beta\ X\ t\ \delta\ \}$$

- Intuition
  - If S is the start symbol then $\$ \in \text{Follow}(S)$
  - If $X \rightarrow A\ B$ then $\text{First}(B) \subseteq \text{Follow}(A)$ and $\text{Follow}(X) \subseteq \text{Follow}(B)$
  - Also if $B \rightarrow^{*} \varepsilon$ then $\text{Follow}(X) \subseteq \text{Follow}(A)$

Computing Follow Sets (Cont.)

Algorithm sketch:

1. Follow(S) $\leftarrow$ { \$ }
2. For each production $A \rightarrow \alpha\ X\ \beta$
   - add First(β) - {ε} to Follow(X)
3. For each $A \rightarrow \alpha\ X\ \beta$ where $\varepsilon \in$ First(β)
   - add Follow(A) to Follow(X)

- repeat step(s) ___ until no Follow set grows

Follow Sets. Example

- Recall the grammar

$E \rightarrow T\ X \qquad\qquad X \rightarrow +\ E \mid \varepsilon$

$T \rightarrow (\ E\ ) \mid \text{int}\ Y \qquad\qquad Y \rightarrow *\ T \mid \varepsilon$

- Follow sets

Follow(+) = { int, (}   Follow(*) = { int, (}

Follow(( ) = { int, (}   Follow(E) = {), $}

Follow(X) = {$,) }   Follow(T) = {+,) , $}

Follow( )) = {+,) , $}   Follow(Y) = {+,) , $}

Follow(int) = {*, +,) , $}

Constructing LL(1) Parsing Tables

- Construct a parsing table T for CFG G
- For each production $A \rightarrow \alpha$ in G do:
  - For each terminal $t \in First(\alpha)$ do
    - $T[A, t] = \alpha$
  - If $\varepsilon \in First(\alpha)$, for each $t \in Follow(A)$ do
    - $T[A, t] = \alpha$
  - If $\varepsilon \in First(\alpha)$ and $\$ \in Follow(A)$ do
    - $T[A, \$] = \alpha$

Notes on LL(1) Parsing Tables

- If any entry is multiply defined then G is not LL(1)
  - If G is ambiguous
  - If G is left recursive
  - If G is not left-factored
  - <u>And in other cases as well</u>
- Most programming language grammars are not LL(1)
- There are tools that build LL(1) tables

Review

- For some grammars there is a simple parsing strategy
  - Predictive parsing

- Next time: a more powerful parsing strategy