



# CS533

## Modeling and Performance Evaluation of Network and Computer Systems

### Experimental Design

(Chapters 16-17)


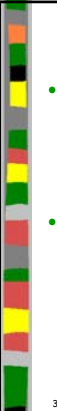
## Introduction (1 of 3)

*No experiment is ever a complete failure. It can always serve as a negative example.*  
– Arthur Bloch

*The fundamental principle of science, the definition almost, is this: the sole test of the validity of any idea is experiment.*  
– Richard P. Feynman

- Goal is to obtain maximum information with minimum number of experiments
- Proper analysis will help separate out the factors
- Statistical techniques will help determine if differences are caused by variations from errors or not



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## Introduction (2 of 3)

- Key assumption is non-zero cost
  - Takes time and effort to gather data
  - Takes time and effort to analyze and draw conclusions
- Minimize number of experiments run
- Good experimental design allows you to:
  - Isolate effects of each input variable
  - Determine effects due to interactions of input variables
  - Determine magnitude of experimental error
  - Obtain maximum info with minimum effort



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## Introduction (3 of 3)

- Consider
  - Vary one input while holding others constant
    - Simple, but ignores possible interaction between two input variables
  - Test all possible combinations of input variables
    - Can determine interaction effects, but can be very large
    - Ex: 5 factors with 4 levels →  $4^5 = 1024$  experiments. Repeating to get variation in measurement error  $1024 \times 3 = 3072$
- There are, of course, in-between choices...
  - (Ch 19, but leads to confounding...)



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## Outline

- Introduction
- **Terminology**
- General Mistakes
- Simple Designs
- Full Factorial Designs
  - $2^k$  Factorial Designs
- $2^{kr}$  Factorial Designs

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## Terminology (1 of 4)

(Will explain terminology using example)

- Study PC performance
  - CPU choice: 6800, z80, 8086
  - Memory size: 512 KB, 2 MB, 8 MB
  - Disk drives: 1-4
  - Workload: secretarial, managerial, scientific
  - Users: high school, college, graduate
- *Response variable* - the outcome or the measured performance
  - Ex: throughput in tasks/min or response time for a task in seconds

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## Terminology (2 of 4)

- **Factors** - each variable that affects response
  - Ex: CPU, memory, disks, workload, user
  - Also called *predictor variables* or *predictors*
- **Levels** - the different values factors can take
  - EX: CPU 3, memory 3, disks 4, workload 3, users 3
  - Also called *treatment*
- **Primary factors** - those of most important interest
  - Ex: maybe CPU and memory the most

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## Terminology (3 of 4)

- **Secondary factors** - of less importance
  - Ex: maybe user type not as important
- **Replication** - repetition of all or some experiments
  - Ex: if run three times, then three replications
- **Design** - specification of the replication, factors, levels
  - Ex: Specify all factors, at above levels with 5 replications so  $3 \times 3 \times 4 \times 3 \times 3 = 324$  time 5 replications yields 1215 total

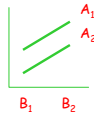
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## Terminology (4 of 4)

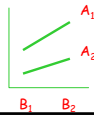
- **Interaction** - two factors A and B interact if one shows dependence upon another
  - Ex: non-interacting factor since A always increases by 2

|                | A <sub>1</sub> | A <sub>2</sub> |
|----------------|----------------|----------------|
| B <sub>1</sub> | 3              | 5              |
| B <sub>2</sub> | 6              | 8              |



- Ex: interacting factors since A change depends upon B

|                | A <sub>1</sub> | A <sub>2</sub> |
|----------------|----------------|----------------|
| B <sub>1</sub> | 3              | 5              |
| B <sub>2</sub> | 6              | 9              |



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- $2^{kr}$  Factorial Designs

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## Common Mistakes in Experiments (1 of 2)

- **Variation due to experimental error is ignored.**
  - Measured values have randomness due to measurement error. Do not assign (or assume) all variation is due to factors.
- **Important parameters not controlled.**
  - All parameters (factors) should be listed and accounted for, even if not all are varied.
- **Effects of different factors not isolated.**
  - May vary several factors simultaneously and then not be able to attribute change to any one.
  - Use of simple designs (next topic) may help but have their own problems.

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## Common Mistakes in Experiments (2 of 2)


- **Interactions are ignored.**
  - Often effect of one factor depend upon another. Ex: effects of cache may depend upon size of program. Need to move beyond one-factor-at-a-time designs
- **Too many experiments are conducted.**
  - Rather than running all factors, all levels, at all combinations, break into steps
  - First step, few factors and few levels
    - Determine which factors are significant
    - Two levels per factor (details later)
  - More levels added at later design, as appropriate

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  - $2^{kr}$  Factorial Designs

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## Simple Designs

- Start with typical configuration
- Vary one factor at a time
- Ex: typical may be PC with z80, 2 MB RAM, 2 disks, managerial workload by college student
  - Vary CPU, keeping everything else constant, and compare
  - Vary disk drives, keeping everything else constant, and compare
- Given  $k$  factors, with  $i$ th having  $n_i$  levels  
 Total =  $1 + \sum (n_i - 1)$  for  $i = 1$  to  $k$
- Example: in workstation study  
 $1 + (3-1) + (3-1) + (4-1) + (3-1) + (3-1) + (3-1) = 14$
- But may ignore interaction (Example next)


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## Example of Interaction of Factors

- Consider response time vs. memory size and degree of multiprogramming


| Degree | 32 MB | 64 MB | 128MB |
|--------|-------|-------|-------|
| 1      | 0.25  | 0.21  | 0.15  |
| 2      | 0.52  | 0.45  | 0.36  |
| 3      | 0.81  | 0.66  | 0.50  |
| 4      | 1.50  | 1.45  | 0.70  |

- If fixed degree 3, mem 64 and vary one at a time, may miss interaction
  - Example: degree 4, non-linear response time with memory

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## Full Factorial Designs


- Every possible combination at all levels of all factors
- Given  $k$  factors, with  $i$ th having  $n_i$  levels  
 Total =  $\prod n_i$  for  $i = 1$  to  $k$
- Example: in CPU design study  
 (3 CPUs)(3 mem) (4 disks) (3 loads) (3 users)  
 = 324 experiments
- Advantage is can find every interaction component
- Disadvantage is costs (time and money), especially since may need multiple iterations (later)
- Can reduce costs by: reduce levels, reduce factors, run fraction of full factorial  
 (Next, reduce levels)

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## $2^k$ Factorial Designs

*Twenty percent of the jobs account for 80% of the resource consumption.*  
 - Pareto's Law

- Very often, many levels at each factor
  - Ex: effect of network latency on user response time  
 → there are lots of latency values to test
- Often, performance continuously increases or decreases over levels
  - Ex: response time always gets higher
  - Can determine direction with min and max
- For each factor, choose 2 alternatives at each level
  - $2^k$  factorial designs
- Then, can determine which of the factors impacts performance the most and study those further

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### 2<sup>2</sup> Factorial Design (1 of 4)

- Special case with only 2 factors
  - Easily analyzed with regression
- Example: MIPS for Mem (4 or 16 Mbytes) and Cache (1 or 2 Kbytes)
 

|            | Mem 4MB | Mem 16MB |
|------------|---------|----------|
| Cache 1 KB | 15      | 45       |
| Cache 2 KB | 25      | 75       |
- Define  $x_a = -1$  if 4 Mbytes mem,  $+1$  if 16 Mbytes
- Define  $x_b = -1$  if 1 Kbyte cache,  $+1$  if 2 Kbytes
- Performance:
 
$$y = q_0 + q_a x_a + q_b x_b + q_{ab} x_a x_b$$

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### 2<sup>2</sup> Factorial Design (2 of 4)

- Substituting:
 
$$15 = q_0 - q_a - q_b + q_{ab}$$

$$45 = q_0 + q_a - q_b - q_{ab}$$

$$25 = q_0 - q_a + q_b - q_{ab}$$

$$75 = q_0 + q_a + q_b + q_{ab}$$
 (4 equations in 4 unknowns)
- Can solve to get:
 
$$y = 40 + 20x_a + 10x_b + 5x_a x_b$$
- Interpret:
  - Mean performance is 40 MIPS, memory effect is 20 MIPS, cache effect is 10 MIPS and interaction effect is 5 MIPS
  - (Generalize to easier method next)

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### 2<sup>2</sup> Factorial Design (3 of 4)

| Exp | a  | b  | y              |
|-----|----|----|----------------|
| 1   | -1 | -1 | Y <sub>1</sub> |
| 2   | 1  | -1 | Y <sub>2</sub> |
| 3   | -1 | 1  | Y <sub>3</sub> |
| 4   | 1  | 1  | Y <sub>4</sub> |

- Solving, we get:
 
$$q_0 = \frac{1}{4}(Y_1 + Y_2 + Y_3 + Y_4)$$

$$q_a = \frac{1}{4}(-Y_1 + Y_2 - Y_3 + Y_4)$$

$$q_b = \frac{1}{4}(-Y_1 - Y_2 + Y_3 + Y_4)$$

$$q_{ab} = \frac{1}{4}(Y_1 - Y_2 - Y_3 + Y_4)$$
- Notice for  $q_a$  can obtain by multiplying "a" column by "y" column and adding
  - Same is true for  $q_b$  and  $q_{ab}$

$$y = q_0 + q_a x_a + q_b x_b + q_{ab} x_a x_b$$

- So:
 
$$Y_1 = q_0 - q_a - q_b + q_{ab}$$

$$Y_2 = q_0 + q_a - q_b - q_{ab}$$

$$Y_3 = q_0 - q_a + q_b - q_{ab}$$

$$Y_4 = q_0 + q_a + q_b + q_{ab}$$

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### 2<sup>2</sup> Factorial Design (4 of 4)

| i | a  | b  | ab | y  |
|---|----|----|----|----|
| 1 | -1 | -1 | 1  | 15 |
| 1 | 1  | -1 | -1 | 45 |
| 1 | -1 | 1  | -1 | 25 |
| 1 | 1  | 1  | 1  | 75 |

| 160 | 80 | 40 | 20 | Total |
|-----|----|----|----|-------|
| 40  | 20 | 10 | 5  | Tt/4  |

- Multiply column entries by  $y_i$  and sum
- Dived each by 4 to give weight in regression model
- Final:
 
$$y = 40 + 20x_a + 10x_b + 5x_a x_b$$
- Column "i" has all 1s
- Columns "a" and "b" have all combinations of 1, -1
- Column "ab" is product of column "a" and "b"

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### Allocation of Variation (1 of 3)

- Importance of a factor measured by proportion of total variation in response explained by the factor
  - Thus, if two factors explain 90% and 5% of the response, then the second may be ignored
    - Ex: capacity factor (768 Kbps or 10 Mbps) versus TCP version factor (Reno or Sack)
- Sample variance of  $y$ 

$$s_y^2 = \frac{\sum(y_i - \bar{y})^2}{(2^2 - 1)}$$
- With numerator being total variation, or Sum of Squares Total (SST)
 
$$SST = \sum(y_i - \bar{y})^2$$

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### Allocation of Variation (2 of 3)


- For a 2<sup>2</sup> design, variation is in 3 parts:
  - SST = 2<sup>2</sup>q<sup>2</sup><sub>a</sub> + 2<sup>2</sup>q<sup>2</sup><sub>b</sub> + 2<sup>2</sup>q<sup>2</sup><sub>ab</sub> (Derivation 17.1, p.287)
- Portion of total variation:
  - of a is 2<sup>2</sup>q<sup>2</sup><sub>a</sub>
  - of b is 2<sup>2</sup>q<sup>2</sup><sub>b</sub>
  - of ab is 2<sup>2</sup>q<sup>2</sup><sub>ab</sub>
- Thus, SST = SSA + SSB + SSAB
- And fraction of variation explained by a:
 
$$= SSA/SST$$
  - Note, may not explain the same fraction of variance since that depends upon errors+

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### Allocation of Variation (3 of 3)

- In the memory-cache study
 
$$\bar{y} = \frac{1}{4} (15 + 55 + 25 + 75) = 40$$
- Total variation
 
$$= \sum (y_i - \bar{y})^2 = (25^2 + 15^2 + 15^2 + 35^2)$$


$$= 2100 = 4 \times 20^2 + 4 \times 10^2 + 4 \times 5^2$$
- Thus, total variation is 2100
  - 1600 (of 2100, 76%) is attributed to memory
  - 400 (of 2100, 19%) is attributed to cache
  - Only 100 (of 2100, 5%) is attributed to interaction
- This data suggests exploring memory further and not spending more time on cache (or interaction) (That was for 2 factors. Extend to k next)



### General 2<sup>k</sup> Factorial Designs (1 of 4)

- Can extend same methodology to k factors, each with 2 levels → Need 2<sup>k</sup> experiments
  - k main effects
  - (k choose 2) two factor effects
  - (k choose 3) three factor effects...
- Can use sign table method

(Show with example, next)




### General 2<sup>k</sup> Factorial Designs (2 of 4)

- Example: design LISP machine
  - Cache, memory and processors

| Factor         | Level -1 | Level 1   |
|----------------|----------|-----------|
| Memory (a)     | 4 Mbytes | 16 Mbytes |
| Cache (b)      | 1 Kbytes | 2 Kbytes  |
| Processors (c) | 1        | 2         |
- The 2<sup>3</sup> design and MIPS perf results are:
 

| Cache (b) | 4 Mbytes Mem(a) |           | 16 Mbytes Mem |           |
|-----------|-----------------|-----------|---------------|-----------|
|           | One proc (c)    | Two procs | One proc      | Two procs |
| 1 KB      | 14              | 46        | 22            | 58        |
| 2 KB      | 10              | 50        | 34            | 86        |




### General 2<sup>k</sup> Factorial Designs (3 of 4)

- Prepare sign table:
 

| i   | a  | b  | c   | ab | ac | bc | abc | y     |
|-----|----|----|-----|----|----|----|-----|-------|
| 1   | -1 | -1 | -1  | 1  | 1  | 1  | -1  | 14    |
| 1   | 1  | -1 | -1  | -1 | -1 | 1  | 1   | 22    |
| 1   | -1 | 1  | -1  | 1  | -1 | -1 | -1  | 10    |
| 1   | 1  | 1  | -1  | 1  | -1 | -1 | -1  | 34    |
| 1   | -1 | 1  | 1   | -1 | -1 | 1  | -1  | 46    |
| 1   | 1  | -1 | 1   | -1 | 1  | -1 | -1  | 58    |
| 1   | -1 | 1  | 1   | -1 | -1 | 1  | -1  | 50    |
| 1   | 1  | 1  | 1   | 1  | 1  | 1  | 1   | 86    |
| 320 | 80 | 40 | 160 | 40 | 16 | 24 | 9   | Ttl   |
| 40  | 10 | 5  | 20  | 5  | 2  | 3  | 1   | Ttl/8 |

$q_a=10, q_b=5, q_c=20$  and  $q_{ab}=5, q_{ac}=2, q_{bc}=3$  and  $q_{abc}=1$




### General 2<sup>k</sup> Factorial Designs (3 of 4)

- $q_a=10, q_b=5, q_c=20$  and  $q_{ab}=5, q_{ac}=2, q_{bc}=3$  and  $q_{abc}=1$
- SST = 2<sup>3</sup> (q<sub>a</sub><sup>2</sup>+q<sub>b</sub><sup>2</sup>+q<sub>c</sub><sup>2</sup>+q<sub>ab</sub><sup>2</sup>+q<sub>ac</sub><sup>2</sup>+q<sub>bc</sub><sup>2</sup>+q<sub>abc</sub><sup>2</sup>)
 
$$= 8 (10^2+5^2+20^2+5^2+2^2+3^2+1^2)$$

$$= 800+200+3200+200+32+72+8$$


$$= 4512$$
- The portion explained by the 7 factors are:
 

|                              |                           |
|------------------------------|---------------------------|
| mem = 800/4512 (18%)         | cache = 200/4512 (4%)     |
| proc = 3200/4512 (71%)       | mem-cache = 200/4512 (4%) |
| mem-proc = 32/4512 (1%)      | cache-proc = 72/4512 (2%) |
| mem-proc-cache = 8/4512 (0%) |                           |



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- 2<sup>k</sup>r Factorial Designs



## 2<sup>kr</sup> Factorial Designs

*No amount of experimentation can ever prove me right; a single experiment can prove me wrong.*  
-Albert Einstein

- With 2<sup>k</sup> factorial designs, not possible to estimate error since only done once
- So, repeat r times for 2<sup>kr</sup> observations
- As before, will start with 2<sup>2r</sup> model and expand
- Two factors at two levels and want to isolate experimental errors
  - Repeat 4 configurations r times
- Gives you error term:
  - $y = q_0 + q_a X_a + q_b X_b + q_{ab} X_a X_b + e$
  - Want to quantify e  
(Illustrate by example, next)

WPI

## 2<sup>2r</sup> Factorial Design Errors (1 of 2)

- Previous cache experiment with r=3

| i   | a    | b   | ab | y            | mean y |
|-----|------|-----|----|--------------|--------|
| 1   | -1   | -1  | 1  | (15, 18, 12) | 15     |
| 1   | 1    | -1  | -1 | (45, 48, 51) | 48     |
| 1   | -1   | 1   | -1 | (25, 28, 19) | 24     |
| 1   | 1    | 1   | 1  | (75, 75, 81) | 77     |
| 164 | 86   | 38  | 20 |              | Total  |
| 41  | 21.5 | 9.5 | 5  |              | Tt/4   |

- Have estimate for each y
  - $Y_i = q_0 + q_a X_{ai} + q_b X_{bi} + q_{ab} X_{ai} X_{bi} + e_i$
- Have difference (error) for each repetition
  - $e_{ij} = Y_{ij} - \bar{y}_i = Y_{ij} - q_0 - q_a X_{ai} - q_b X_{bi} - q_{ab} X_{ai} X_{bi}$

WPI

## 2<sup>2r</sup> Factorial Design Errors (2 of 2)

- Use sum of squared errors (SSE) to compute variance and confidence intervals

$$SSE = \sum \sum e_{ij}^2 \text{ for } i = 1 \text{ to } 4 \text{ and } j = 1 \text{ to } r$$

- Example

| i | a  | b  | ab | Y <sub>i</sub> | Y <sub>i1</sub> | Y <sub>i2</sub> | Y <sub>i3</sub> | e <sub>i1</sub> | e <sub>i2</sub> | e <sub>i3</sub> |
|---|----|----|----|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1 | -1 | -1 | 1  | 15             | 15              | 18              | 12              | 0               | 3               | -3              |
| 1 | 1  | -1 | -1 | 48             | 45              | 48              | 51              | -3              | 0               | 3               |
| 1 | -1 | 1  | -1 | 24             | 25              | 28              | 19              | 1               | 4               | -5              |
| 1 | 1  | 1  | 1  | 77             | 75              | 75              | 81              | -2              | -2              | 4               |

- Ex:  $y_1 = q_0 - q_a - q_b + q_{ab} = 41 - 21.5 - 9.5 + 5 = 15$
- Ex:  $e_{11} = Y_{11} - \bar{y}_1 = 15 - 15 = 0$
- $SSE = 0^2 + 3^2 + (-3)^2 + (-3)^2 + 0^2 + 3^2 + 1^2 + 4^2 + (-5)^2 + (-2)^2 + (-2)^2 + 4^2 = 102$

WPI

## 2<sup>2r</sup> Factorial Allocation of Variation

- Total variation (SST)
 
$$SST = \sum (y_{ij} - \bar{y})^2$$
- Can be divided into 4 parts:
 
$$\sum (y_{ij} - \bar{y})^2 = 2^2 r q_a^2 + 2^2 r q_b^2 + 2^2 r q_{ab}^2 + \sum e_{ij}^2$$

$$SST = SSA + SSB + SSAB + SSE$$
- Thus
  - SSA, SSB, SSAB are variations explained by factors a, b and ab
  - SSE is unexplained variation due to experimental errors
- Can also write  $SST = SSY - SS0$  where SS0 is sum squares of mean

(Derivation 18.1, p.296) WPI

## 2<sup>2r</sup> Factorial Allocation of Variation Example

- For memory cache study:
  - $SSY = 15^2 + 18^2 + 12^2 + \dots + 75^2 + 81^2 = 27,204$
  - $SS0 = 2^2 r q_0^2 = 12 \times 41^2 = 20,172$
  - $SSA = 2^2 r q_a^2 = 12 \times (21.5)^2 = 5547$
  - $SSB = 2^2 r q_b^2 = 12 \times (9.5)^2 = 1083$
  - $SSAB = 2^2 r q_{ab}^2 = 12 \times 5^2 = 300$
  - $SSE = 27,204 - 2^2 \times 3(41^2 + 21.5^2 + 9.5^2 + 5^2) = 102$
  - $SST = 5547 + 1083 + 300 + 102 = 7032$
- Thus, total variation of 7032 divided into 4 parts:
  - Factor a explains 5547/7032 (78.88%), b explains 15.40%, ab explains 4.27%
  - Remaining 1.45% unexplained and attributed to error

WPI


## Confidence Intervals for Effects

- Assuming errors are normally distributed, then  $y_{ij}$ 's are normally distributed with same variance
- Since  $q_0, q_a, q_b, q_{ab}$  are all linear combinations of  $y_{ij}$ 's (divided by 2<sup>2r</sup>), then they have same variance (divided by 2<sup>2r</sup>)
- Variance  $s^2 = SSE / (2^2(r-1))$
- Confidence intervals for effects then:
  - $q_i \pm t_{[1-\alpha/2; 2^2(r-1)]} S_{qi}$
- If confidence interval does not include zero, then effect is significant

WPI


### Confidence Intervals for Effects (Example)

- Memory-cache study, std dev of errors:  
 $s_e = \sqrt{\text{SSE} / (2^2(r-1))} = \sqrt{102/8} = 3.57$
- And std dev of effects:  
 $s_{q_i} = s_e / \sqrt{2^2r} = 3.57/3.47 = 1.03$
- The t-value at 8 degrees of freedom and 95% confidence is 1.86
- Confidence intervals for parameters:  
 $q_i \pm (1.86)(1.03) = q_i \pm 1.92$ 
  - $q_0 \rightarrow (39.08, 42.91)$ ,  $q_a \rightarrow (19.58, 23.41)$ ,  
 $q_b \rightarrow (7.58, 11.41)$ ,  $q_{ab} \rightarrow (3.08, 6.91)$
  - Since none include zero, all are statistically significant




### Confidence Intervals for Predicted Responses (1 of 2)

- Mean response predicted  
 $Y = q_0 + q_a X_a + q_b X_b + q_{ab} X_a X_b$
- If predict mean from  $m$  more experiments, will have same mean but confidence interval on predicted response decreases
- Can show that std dev of predicted  $y$  with  $m$  more experiments  
 $s_{y_m} = s_e \sqrt{1/n_{\text{eff}} + 1/m}$ 
  - Where  $n_{\text{eff}} = \text{runs}/(1+\text{df})$ 
    - In 2 level case, each parameter has 1 df, so  $n_{\text{eff}} = 2^2r/5$




### Confidence Intervals for Predicted Responses (2 of 2)

- A  $100(1-\alpha)\%$  confidence interval of response:  
 $Y_p \pm t_{[1-\alpha/2; 2^2(r-1)]} s_{y_m}$
- Two cases are of interest.
  - Std dev of one run ( $m=1$ )  
 $s_{y_1} = s_e \sqrt{5/2^2r + 1}$
  - Std dev for many runs ( $m=\infty$ )  
 $s_{y_1} = s_e \sqrt{5/2^2r}$



### Confidence Intervals for Predicted Responses Example (1 of 2)

- Mem-cache study, for  $x_a=-1, x_b=-1$
- Predicted mean response for future experiment  
 $Y_1 = q_0 - q_a - q_b + q_{ab} = 41 - 21.5 + 1 = 15$ 
  - Std dev =  $3.57 \times \sqrt{5/12 + 1} = 4.25$
- Using  $t[0.95; 8] = 1.86$ , 90% conf interval  
 $15 \pm 1.86 \times 4.25 = (8.09, 22.91)$
- Predicted mean response for 5 future experiments  
 $Y_5 = 15$ 
  - Std dev =  $3.57 \sqrt{5/12 + 1/5} = 2.80$
  - 90% conf interval:  $15 \pm 1.86 \times 2.80 = (9.79, 20.29)$



### Confidence Intervals for Predicted Responses Example (2 of 2)

- Predicted Mean Response for Large Number of Experiments  
 $Y_5 = 15$ 
  - Std dev =  $3.57 \times \sqrt{5/12} = 2.30$
  - The confidence interval:  
 $15 \pm 1.86 \times 2.30 = (10.72, 19.28)$

