QUIZ 6

a Describe a Binomial Heap implementation of a (min-)priority queue and a sequence of (eager) INSERTions and EXTRACT-MINS such that each operation in the sequence requires time in $\Theta(\lg n)$.

SOLUTION: Let the binomial heap start with $n = 2^k - 1$ elements, for some $k$. A sequence of

$$\text{INSERT}(x,H), \text{EXTRACT-MIN}(H), \text{INSERT}(x,H), \text{EXTRACT-MIN}(H), \text{INSERT}(x,H), \text{EXTRACT-MIN}(H),...$$

operations will each take time in $\Theta(\lg n)$, since $H$ has $\Theta(\lg n)$ binomial trees.

b Prim’s Algorithm works by starting at a vertex and growing a tree by continually adding to the tree the lightest edge out of the tree. Prove that it constructs an MST. Hint: Think Blue Rule and Red Rule.

SOLUTION: The cut between the vertices of the tree being constructed and the rest of the vertices has no edges of the tree, no blue edges. So a lightest edge in the cut can be colored blue. Continuing $|V|-1$ times we know that we finish with an MST.