QUIZ 3

For fixed \( k \geq 1 \) (such as \( k = 423 \) or \( k = 1 \)) and arbitrarily large \( n \), describe an optimal algorithm to find the \( k \)th largest element of array \( A[1..n] \), and show that your algorithm is asymptotically optimal.

**Solution:**

Build-Max-Heap\((A)\) \( O(n) \)

\[
\text{for } i \leftarrow 1 \text{ to } k-1 \text{ do } \text{EXTRACT-MAX}(A) \quad O(k \lg n)
\]

\[
\text{return } A[1] \quad O(1)
\]

For \( k = 1 \) the algorithm takes time linear (in \( n \)), which we showed to be optimal. We established the lower bound in class that finding the 2nd largest element of \( A \) requires at least \( n + \lg n - 2 \in \Omega(n) \) operations, and for fixed \( k \) the above algorithm takes \( O(n + k \lg n) = O(n) \) operations.