In the **STABLE MARRIAGE PROBLEM** each of \( n \) men ranks (in a total order) all of \( n \) women, and each of \( n \) women ranks (in a total order) all of \( n \) men.

**a** Describe a necessary and sufficient set of conditions on the \( 2n \) rankings such that each woman marrying her first choice man is a stable marriage.

**SOLUTION:** Each woman marrying her first choice is a stable marriage if and only if each woman’s first choice is different from every other woman’s first choice. That is, the women’s first choices are a permutation of the men. This condition is necessary because if two women have the same first choice then they can’t both marry him. The condition is sufficient because if each woman marries her first choice, she will never want to break the marriage for another man.

**b** Assuming that each woman’s choices are drawn from uniform distribution over the set of permutations of men, and each man’s choices are drawn from uniform distribution over the set of permutations of women, what is the probability that having each woman marry her first choice yields a stable marriage?

**SOLUTION:** Ordering the women \( w_1, w_2, ..., w_n \), the probability that \( w_i \)’s first choice is different from the first choices of \( w_1, ..., w_{i-1} \) is \( \frac{n-1}{n} \times \frac{n-i+1}{n} \), so the probability that all of the \( n \) women’s first choices are distinct is \( \frac{n!}{n^n} \).