1. (40 points) For each of the following languages over \{0, 1\}, tell whether or not it is regular. Justify each answer.

   \( a \) \( L \) is the set of strings which contain either 0010 or 110 and do not contain an instance of 11011. So 11011000 \( \notin \) \( L \) and 110010 \( \in \) \( L \), but \( \varepsilon \) \( \notin \) \( L \) and 011011 \( \notin \) \( L \).
$bL$ is the set of strings with twice as many 0's as 1's. So $11001000 \in L$ and $\varepsilon \in L$, but $00110 \notin L$. 
2. (30 points) Let $L$ be the set of all binary strings of odd length with middle symbol 0.

$a$ Is $L$ regular? Justify your answer.

$b$ Is $L$ context-free? Justify your answer.
3. (30 points) Assume that $L$ is a set of binary strings. Define a new language

$$\Lambda(L) = \left\{ z \left| (z \cup \{0,1\}^*) \land (\exists w \in L) |w| = |z| \right. \right\}.$$  

That is, $\Lambda(L)$ is the set of all binary strings such that $L$ contains a binary string of the same length. Either give a counterexample to the following or give a construction suggesting that it must be true.

**Conjecture:** For any regular language $L$, $\Lambda(L)$ must be regular.
1. \(a\) L is regular. The set of strings which contain 0010 is described by the regular
expression \((0+1)^*0010(0+1)^*\). The set of strings which contain 110 is described by the
regular expression \((0+1)^*110(0+1)^*\). Because regular languages are closed under union,
the set of strings which contain either 0010 or 110 is regular. The set of strings which
contain 11011 is described by the regular expression \((0+1)^*11011(0+1)^*\). Because
regular languages are closed under complement, the set of strings which do not contain
11011 is regular. Because regular languages are closed under intersection, the set of
strings which contain either 0010 or 110 and do not contain an instance of 11011 is
regular.

\(b\) We show that the set of strings with twice as many 0's as 1's is not regular by invoking
the Pumping Lemma for regular languages. We assume that \(L\) is regular and we let \(k\) be
the constant assured by the Pumping Lemma. Choose \(z = 1^k0^2^k\). If \(z\) is expressed as \(uvw\)
with \(|uv| \leq k\) and \(|v| \geq 1\) then string \(v\) contains at least one 1 and does not contain any 0's.
The Pumping Lemma assures us that \(uv^2w \in L\). But the number of 0's in \(uv^2w\) is less than
2 times the number of 1's, so \(uv^2w \notin L\). This contradiction means that \(L\) cannot be regular.

2. \(a\) We show that \(L\) is not regular by invoking the Pumping Lemma. Let \(n\) be the constant
assured by the PL, and let \(z = 1^n0^n\). For any decomposition of \(z\) into \(uvw\) with \(|uv| \leq n\)
and \(|v| \geq 1\), it must be the case that \(v\) is a nonempty sequence of 1's. The PL says that
\(uv^2w \in L\), but \(uv^2w = 1^n0^n1^m, m > n\), so the middle symbol is a 1 and \(uv^2w \notin L\). By this
contradiction, \(L\) can not be regular.

\(b\) \(L\) is context free. It is generated by the grammar

\[
S \rightarrow ASA10 \\
A \rightarrow 011
\]

3. The Conjecture is true. If \(L\) is regular, then there is a DFA \((Q,\{0,1\},\Delta,s,F)\) to accept it.
Language \(\Lambda(L)\) is accepted by the NFA \((Q,\{0,1\},\Delta,\{s\},F)\), where
\(\Delta(q,a) = \{\delta(q,0),\delta(q,1)\}\) for all \(q \in Q, a \in \{0,1\}\).