1. (6 points) Give a CFG to generate each of the following languages.
   \( a \) \( \{a^ib^i\} \).
   \( b \) \( \{a^ib^ic^i|j, k \geq 0\} \).
   \( c \) \( \{a^ib^ic^{2ij} | i, j \geq 0\} \).

2. (8 points) Prove that \( \{0^m1^n|10 \leq m < n\} \) is the language generated by
   \[
   S \rightarrow 0S11A \\
   A \rightarrow 1A11
   \]

3. (12 points) A derivation is nontrivial if it contains at least one step. In a CFG
   \((N, \Sigma, P, S)\), a nonterminal \( A \in N \) is recursive if there is a nontrivial derivation
   \( A^+ \Rightarrow uAv, u, v \in (N \cup \Sigma)^* \).

Tell whether or not each of the following Conjectures is true and justify your responses.

**Conjecture A:** For any CFL \( L \), there is a CFG \( G=(N, \Sigma, P, S) \) such that \( L = L(G) \) and
\( S \) is not recursive.

**Conjecture B:** For any CFG \( G=(N, \Sigma, P, S) \) such that \( A \rightarrow w, w \in (N \cup \Sigma)^* \), if
\( G^* = (N, \Sigma, P \cup \{A \rightarrow w\}, S) \), then \( L(G) = L(G^*) \).

**Conjecture C:** For any CFG \( G=(N, \Sigma, P, S) \) if \( A \rightarrow uBv \in P, A, B \in N \) and
\( B \rightarrow w_1|w_2|...|w_n \) is the set of productions with \( B \) on the left side, and if
\( G^* = (N, \Sigma, P - \{A \rightarrow uBv\} \cup \{A \rightarrow uw_1v1uw_2v1...1uw_nv\}, S) \), then \( L(G) = L(G^*) \).