1. (8 points) Remove useless states from the NFA $N = (\{s, q_0, q_1, q_2\}, \{0, 1\}, \Delta, \{s\}, \{q_1, q_2\})$ where

$$
\begin{array}{ccc}
\Delta & 0 & 1 \\
s & \{q_0, q_2\} & \emptyset \\
q_0 & \emptyset & \emptyset \\
q_1 & \emptyset & \{q_2\} \\
q_2 & \{q_2\} & \emptyset
\end{array}
$$

and then use the algorithm developed in class to construct a regular expression $\alpha$ such that $L(N) = L(\alpha)$.

2. (8 points) Is there a decision procedure which accepts as input two NFAs $M_0, M_1$ and decides whether $L(M_0) = L(M_1)$? Justify your answer.
1. We first note that both $q_0$ and $q_1$ are useless. We then follow the algorithm to construct regular expressions:

$$
\alpha_{s,s} = \varepsilon
$$

$$
\alpha_{s,q_2} = 0
$$

$$
\alpha_{q_2,s} = \emptyset
$$

$$
\alpha_{q_2,q_2} = 0 + \varepsilon
$$

$$
\alpha_{s,s} = \varepsilon
$$

$$
\alpha_{s,q_2} = 0 + \varepsilon
$$

$$
\alpha_{q_2,s} = \emptyset
$$

$$
\alpha_{q_2,q_2} = 0 + \varepsilon
$$

$$
\alpha_{s,q_2}^{\{s, q_2\}} = \emptyset
$$

$$
\alpha_{s,s}^{\{s, q_2\}} = \varepsilon
$$

$$
\alpha_{s,q_2}^{\{s, q_2\}} = \varepsilon
$$

$$
\alpha_{s,q_2}^{\{s, q_2\}} = 0 + \varepsilon + (0 + \varepsilon)(0 + \varepsilon)^* (0 + \varepsilon)
$$

$$
\alpha_{q_2,s}^{\{s, q_2\}} = \emptyset
$$

$$
\alpha_{q_2,q_2}^{\{s, q_2\}} = 0^*
$$

So the regular expression which is returned is

$$
0 + \varepsilon + (0 + \varepsilon)(0 + \varepsilon)^* (0 + \varepsilon).
$$

2. There is such a decision procedure.

$$
(L(M_0) = L(M_1)) \Leftrightarrow (L(M_0) \cap \overline{L(M_1)}) \cup (L(M_1) \cap \overline{L(M_0)}) = \emptyset.
$$

Because regular languages are closed under union, intersection and complement, for any DFAs $M_0, M_1$ we can construct a DFA $M^*$ to accept

$$
(L(M_0) \cap \overline{L(M_1)}) \cup (L(M_1) \cap \overline{L(M_0)}).
$$

So $L(M_0) = L(M_1)$ if and only if $L(M^*) = \emptyset$. 