1. (6 points) For the CFG $G$

$$
S \rightarrow S + T | T \\
T \rightarrow T * F | F \\
F \rightarrow (S) | x
$$

and for each of the following strings, give as many parse trees as possible for the string to show that it belongs to $L(G)$.

- $a$  $x$
- $b$  $x + x + x$
- $c$  $x * ((x)) * x$

2. (8 points) Show that $\{(01)^i 0^j \mid i > j \geq 0\}$ is not regular.

3. (16 points) a Is the language $\{a^i b^j c^k \mid 0 \leq i, 0 \leq j \leq k\}$ regular? Justify your answer.
   b Prove that $\{a^i b^j c^k \mid 0 \leq i, 0 \leq j \leq k\}$ is a CFL by providing a CFG without $\varepsilon$–productions and unit productions to generate $\{a^i b^j c^k \mid 0 \leq i, 0 \leq j \leq k\} - \{\varepsilon\}$. 

1. \(a\)

\[
\begin{array}{c}
S \\
F \\
x
\end{array}
\]

\(b\)

\[
\begin{array}{c}
S \\
+ \\
S' \\
+ \\
T \\
F \\
x
\end{array}
\]

\(c\)

\[
\begin{array}{c}
S \\
+ \\
S' \\
+ \\
T \\
F \\
x
\end{array}
\]

2. Assume \(\{(01)^i 0^i | i > j \geq 0\}\) is regular. Let \(n\) be the number guaranteed by the Pumping Lemma. Choose \((01)^{n+1} 0^n \in \{(01)^i 0^i | i > j \geq 0\}\). Any decomposition of \((01)^{n+1} 0\) into \(uvw\) with \(|uv| \leq n\) and \(|v| \geq 1\) has \(v\) be a nonempty part of \((01)^{n+1}\). If \(v=0\) or \(v=1\), then \(uv^0 w \not\in \{(01)^i 0^i | i > j \geq 0\}\). Likewise, if \(|v| > 1\) then \(uv^2 w \not\in \{(01)^i 0^i | i > j \geq 0\}\). So by contradiction \(\{(01)^i 0^i | i > j \geq 0\}\) can not be regular.

3. \(a\) Assume \(\{a^i b^i c^k | 0 \leq i, 0 \leq j \leq k\}\) is regular, and let \(n\) be the number guaranteed by the Pumping Lemma. Choose \(b^{n+1} c^{n+1} \in \{a^i b^i c^k | 0 \leq i, 0 \leq j \leq k\}\). Any decomposition of \(b^{n+1} c^{n+1}\)
into $uvw$ with $|uv| \leq n$ and $|v| \geq 1$ has $v$ being a nonempty sequence of $bs$. “Pumping up” the $bs$, the Pumping Lemma assures us that $uv^2w \in \{a^ib^j c^k | 0 \leq i, 0 \leq j \leq k \}$. But $uv^2w$ has more $bs$ than $cs$. This contradiction yields that $\{a^ib^j c^k | 0 \leq i, 0 \leq j \leq k \}$ can not be regular.

**b** We first provide a CFG to generate $\{a^ib^j c^k | 0 \leq i, 0 \leq j \leq k \}$.

$$
S \rightarrow AB \\
A \rightarrow aA | \varepsilon \\
B \rightarrow CD \\
C \rightarrow bCc | \varepsilon \\
D \rightarrow cD | \varepsilon 
$$

We then remove $\varepsilon$–productions by first computing $\text{NULLABLE} = N = \{S, A, B, C, D\}$. Then we construct the following CFG without $\varepsilon$–productions to generate $\{a^ib^j c^k | 0 \leq i, 0 \leq j \leq k \} - \{\varepsilon \}$.

$$
S \rightarrow AB | A | B \\
A \rightarrow aA | a \\
B \rightarrow CD | C | D \\
C \rightarrow bCc | bc \\
D \rightarrow cD | c 
$$

We then remove unit productions to the following CFG without $\varepsilon$–productions or unit productions to generate $\{a^ib^j c^k | 0 \leq i, 0 \leq j \leq k \} - \{\varepsilon \}$.

$$
S \rightarrow AB | aA | a | CD | bCc | bc | cD | c \\
A \rightarrow aA | a \\
B \rightarrow CD | bCc | bc | cD | c \\
C \rightarrow bCc | bc \\
D \rightarrow cD | c 
$$