1. (6 points) Give a regular expression for the following languages over \( \{0,1\} \).

   a The set of strings that contain 01 and 10. Note that 010 belongs to the language.
   b Every 0 is either immediately preceded by a 1 or immediately followed by a 1.

2. (8 points) For the languages described by the regular expressions
   \[ \alpha = 0^* + 1^* \]
   \[ \beta = 01^* + 10^* + 1^0 + \left(0^1\right)^* \]
   find a shortest string \( z \) satisfying each of the following conditions.

   a \( z \in L(\alpha) \) and \( z \notin L(\beta) \).
   b \( z \notin L(\alpha) \) and \( z \in L(\beta) \).
   c \( z \in L(\alpha) \cap L(\beta) \).
   d \( z \notin L(\alpha) \cup L(\beta) \).

3. (6 points) Letting \( \alpha \) and \( \beta \) denote arbitrary regular expressions, find a shortest regular expression which is equivalent to each of the following.

   a \( \alpha (\alpha + \alpha^*) + \alpha^* \)
   b \( (\alpha + \varepsilon)^* \)
   c \( (\alpha + \beta)^* \alpha \beta (\alpha + \beta)^* + \beta^* \alpha^* \)

4. (10 points) Describe an algorithm to convert an arbitrary regular expression \( \alpha \), such that \( L(\alpha) \neq \emptyset \), to an equivalent regular expression \( \beta \) such that \( L(\alpha) = L(\beta) \) and \( \emptyset \) does not appear in \( \beta \).
CS3133
Solutions to HW#3

1. \( a \) Either the first 01 precedes the first 10 or the first 10 precedes the first 01.
   \[ 0^*11^*0(0+1)^* + 1^*00^*1(0+1)^* \]

   \( b \) \( 1^*(101^*+1^*01)^*1^* \)

2. \( a \) 00
   \( b \) 01
   \( c \) \( \varepsilon \)
   \( d \) 010

3. \( a \) \( \alpha^* \)
   \( b \) \( \alpha^* \)
   \( c \) \( (\alpha + \beta)^* \)

4. \( \beta \leftarrow \alpha \)
   \( \text{repeat} \)
   \( \text{if} \ \varepsilon + \chi \ \text{or} \ \chi + \varepsilon \ \text{appears in} \ \beta \ \text{for some regular expression} \ \chi \)
   \( \text{then replace it by} \ \chi \)
   \( \text{if} \ \varepsilon \chi \ \text{or} \ \chi \varepsilon \ \text{appears in} \ \beta \ \text{for some regular expression} \ \chi \)
   \( \text{then replace it by} \ \varepsilon \)
   \( \text{if} \ \varepsilon^* \ \text{appears in} \ \beta \ \text{then replace it by} \ \varepsilon \)
   \( \text{until} \ \varepsilon \ \text{does not appear in} \ \beta \)