1. (6 points) Give a regular expression for the set of all strings over \( \{a, b\} \) which contain at least two \( a \)'s and at least one \( b \).

2. (12 points) For any language \( L \), we define a new language \( \text{Prefix}(L) \) to be \( \{ y | \exists z \ y z \in L \} \). So \( \text{Prefix}(\{011, 1\}) = \{\varepsilon, 0, 01, 011, 1\} \). Describe an algorithm to accept as input a regular expression \( \alpha \) and to construct a regular expression for \( \text{Prefix}(L(\alpha)) \). So upon receiving input 011+1 your algorithm would return \( \varepsilon + 0 + 01 + 011 + 1 \), or an equivalent regular expression.

3. (6 points) Prove or give a counterexample to the following.

   **CONJECTURE:** Any subset of a regular set is regular.

4. (20 points) Are the following languages regular? Defend your answers.
   a \( \{0^i1^j w | i \geq 0 \land w \in \{0,1\}^*\} \)
   b \( \{a^m b^n c^l | l > m > n \geq 0\} \).
   c \( \{0^m1^n | m \neq n\} \) Hint: Express it in terms of other languages we have seen.
   d \( LL \) for all regular languages \( L \).

5. (3 points) Give a regular expression for the language of assignment statements such as \( E \leftarrow 2.71828 \) or \( PI \leftarrow 3.14159265358979323 \) An assignment statement is a nonempty string of uppercase letters, followed by a left arrow, "\( \leftarrow \)" , followed by a nonempty string of digits without a leading 0, followed by a decimal point, ",." , followed by a nonempty string of digits without a trailing 0.

6. (8 points) Do **PROBLEM 14** on page 320 of Kozen.

7. (4 points) Let \( \alpha = (00 + 11)^* (01 + 10)(00 + 11)^* \).
   a Give an NFA \( N \) such that \( L(N) = L(\alpha) \).
   b Give a \( w \in \{0,1\}^* \), \(|w| = 8\) such that \( w \not\in L(\alpha) \).
1. \((a+b)^* aab(a+b)^*(a+b)^* baa(a+b)^* (a+b)^* a(a+b)^*\)

2. One solution is to apply the function \(\text{Prefix}\) recursively to regular expression \(\alpha\).
\[
\begin{align*}
\alpha & \quad \text{Prefix}(\alpha) \\
\emptyset & \quad \emptyset \\
a \in \Sigma & \quad a + \varepsilon \\
\alpha + \beta & \quad \text{Prefix}(\alpha) + \text{Prefix}(\beta) \\
\alpha\beta & \quad \text{Prefix}(\alpha) + \alpha \text{Prefix}(\beta) \\
\alpha^* & \quad \alpha^* \text{Prefix}(\alpha)
\end{align*}
\]

A different solution is to use THEOREM 8.1 of the text to convert \(\alpha\) to a DFA \(M\) such that \(L(\alpha) = L(M)\), then convert \(M\) to another DFA \(M'\) which is equivalent to \(M\) except that every state of \(M\) from which a final state of \(M\) is reachable becomes a final state of \(M\).
\[\text{Prefix}(L(\alpha)) = \text{Prefix}(L(M)) = L(M').\]
Finally use THEOREM 8.1 to construct a regular expression \(\beta\) such that \(L(\beta) = L(M') = \text{Prefix}(L(\alpha))\).

3. The CONJECTURE is false. \(\{0,1\}^*\) is regular, because it is accepted by \(\{\{q\},\{0,1\},\delta,q,\{q\}\}\) with \(\delta(q,0) = \delta(q,1) = q\). But we showed that \(\{0^n1^n \mid n \geq 0\}\) isn't regular, even though \(\{0^n1^n \mid n \geq 0\} \subseteq \{0,1\}^*\).

4. a \(\{0^i1^j \mid i \geq 0 \land w \in \{0,1\}^*\} = \{0,1\}^*\), so it is regular. It is accepted by the DFA \(\{\{q\},\{0,1\},\delta,q,\{q\}\}\) with \(\delta(q,0) = \delta(q,1) = q\).

b \(\{a^m b^n c^m \mid l > m > n \geq 0\}\) is not regular. If it were, the Pumping Lemma assures us of the existence of a \(k\) such that \(a^{k+1}b^k c^l\) can be expressed as \(xyz\) such that \(\{xy'z \mid i \geq 0\} \subseteq \{a^m b^n c^m \mid l > m > n \geq 0\}\). There are 5 cases to consider:

- case: \(y\) spans only \(a's\). \(xy^0z\) does not have more \(a's\) than \(b's\), so this is a contradiction.
- case: \(y\) spans some \(a's\) and some \(b's\). \(xy^2z\) has \(a's\) followed by \(b's\) followed by \(a's\) followed by \(b's\), so this is a contradiction.
- case: \(y\) spans only \(b's\). \(xy^0z\) does not have more \(b's\) than \(c's\), so this is a contradiction.
- case: \(y\) spans some \(b's\) and some \(c's\). \(xy^2z\) has \(b's\) followed by \(c's\) followed by \(b's\) followed by \(c's\), so this is a contradiction.
- case: \(y\) spans only \(c's\). \(xy^2z\) does not have more \(b's\) than \(c's\), so this is a contradiction.
Since each of the five possible cases yields a contradiction, \( \{a^ib^ic^n \mid l > m > n \geq 0 \} \) is not regular.

c \{0^n1^* \mid m \neq n \} is not regular. PROOF: We know that the language \( L(0^1^*) \) is regular and that regular languages are closed under set difference. If \( \{0^n1^* \mid m \neq n \} \) were regular, then the fact that \( \{0^1^* \mid n \geq 0 \} = L(0^1^*) - \{0^n1^* \mid m \neq n \} \) would contradict our previous result that \( \{0^n1^* \mid n \geq 0 \} \) is not regular.

d As we saw in class and on the top of page 22 of our text, the regular languages are closed under concatenation.

5. \((A+...+Z)(A+...+Z)^* \leftarrow (1...9)(0...9)^*(0...9)^*(1...9)\)

6.

DFA \( \left( \{\{a\}, \{b\}, \{a,c\}, \{a,d\}, \emptyset \}, \{0,1\}, \delta, \{a\}, \emptyset \right) \)

\[
\begin{array}{ccc}
0 & 1 \\
\{a\} & \{b\} & \emptyset \\
\{b\} & \emptyset & \{a,c\} \\
\{a,c\} & \{b\} & \{a,d\} \\
\{a,d\} & \{b\} & \{a\} \\
\emptyset & \emptyset & \emptyset \\
\end{array}
\]
\textbf{7. a}

\[ b \ w = 00000000 \]