1. (9 points) \(a\) Describe a three state DFA to accept all binary strings that end with 00.
\(b\) Describe a DFA to accept all binary strings that don’t contain an 011.
\(c\) Describe a four state DFA to accept all binary strings that contain an odd number of 0’s and an even number of 1’s.

2. (10 points) For \(a_1a_2\ldots \in \Sigma^*\), we define \(\text{even}(a_1a_2\ldots) = a_2a_4a_6\ldots\) So \(\text{even}(0110101) = \text{even}(011010) = 100\) and \(\text{even}(0) = \text{even}(\varepsilon) = \varepsilon\). Show that if \(L\) is regular, then so is \(\{\text{even}(z) \mid z \in L\}\).

3. (6 points) Show that any regular language admits an infinite number of nonisomorphic DFAs to accept it. Two DFAs \(M_0 = (Q_0, \Sigma, \delta_0, s_0, F_0)\) and \(M_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)\) are isomorphic if they are identical up to renaming of the states. That is, they are isomorphic if there exists a bijection \(f : Q_0 \to Q_1\) such that \(f(s_0) = s_1\) and \(F_1 = \{f(q) \mid q \in F_0\}\) and \((\forall q \in Q_0)(\forall a \in \Sigma)f(\delta_0(q,a)) = \delta_1(f(q),a)\).
1. \( a \) \( [\{s,p,q\},\{0,1\},\delta, s, \{q\}] \) where
\[
\delta \\
0 & 1 \\
s & p & s \\
p & q & s \\
q & q & s \\
\]

\( b \) \( [\{s,p,q,r\},\{0,1\},\delta, s, \{s,p,q\}] \) where
\[
\delta \\
0 & 1 \\
s & p & s \\
p & p & q \\
q & p & r \\
r & r & r \\
\]

\( c \) In the DFA, state \( q_{\text{even,odd}} \) "means" that the input string which has been consumed contained an even number of 0's and an odd number of 1's. The other three states can be "defined" analogously. The language is accepted by
\[
[\{q_{\text{even,even}}, q_{\text{even,odd}}, q_{\text{odd,even}}, q_{\text{odd,odd}}\}, \{0,1\}, \delta, q_{\text{even,even}}, \{q_{\text{even,odd}}\}] \\
\]
where
\[
\delta \\
0 & 1 \\
q_{\text{even,even}} & q_{\text{even,odd}} & q_{\text{even,odd}} \\
q_{\text{even,odd}} & q_{\text{odd,even}} & q_{\text{even,even}} \\
q_{\text{odd,even}} & q_{\text{even,even}} & q_{\text{odd,odd}} \\
q_{\text{odd,odd}} & q_{\text{even,odd}} & q_{\text{odd,even}} \\
\]

2. Let \( N = (Q, \Sigma, \Delta, S, F) \) be an NFA to accept \( L \). Language \( \{\text{even}(z) \mid z \in L\} \) is accepted by \( N^* = (Q, \Sigma, \Delta^*, S, F) \) where for all \( q \in Q, a \in \Sigma \)
\[
\Delta^*(q,a) = \bigcup_{p \in \Delta(q,a)} \bigcup_{b \in \Sigma} \Delta(p,a) \\
\]

3. If any language over \( \Sigma \) is regular, there is a DFA \( M = (Q, \Sigma, \delta, S, F) \) to accept it. For any \( i > 0 \), we add \( i \) new states \( p_1,\ldots,p_i \not\in Q \) which are not reachable from \( Q \). The language \( L(M) \) is accepted by \( M_i = (Q \cup \{p_1,\ldots,p_i\}, \Sigma, \delta, S, F) \), where
\[
(\forall q \in Q)(\forall a \in \Sigma)\delta_1(q,a) = \delta(q,a), \text{ and } (\forall j, 1 \leq j \leq i)(\forall a \in \Sigma)\delta(p_j,a) = p_j. \text{ If } i \neq j, \text{ then } M_i \text{ is nonisomorphic to } M_j, \text{ and the list of machines } M, M_1, M_2, \ldots \text{ is infinite.}