1. (8 points) Let $A = \{00, 11\}$ and $B = \{\epsilon, 1, 01\}$.
   
   a List the strings in $AB$.
   
   b What is the cardinality of $\{\alpha \mid \alpha \in A^* \land |\alpha| = 6\}$?
   
   c List the members of $\{\alpha \mid \alpha \in B^* \land |\alpha| \leq 3\}$.
   
   d List the members of $\{\alpha \mid \alpha \in A^*B^* \land |\alpha| \leq 4\}$.

2. (5 points) Let $L_1 = \{000\}^*$, $L_2 = \{0, 1\} \{0, 1\} \{0, 1\} \{0, 1\}$ and $L_3 = L_2^*$. Describe $L_1 \cap L_3$.
   
   What is the cardinality of $L_1 \cap L_3$?

3. (7 points) Show that the language $L = \left\{ x_1...x_n \mid x_1...x_n \in \{0, 1, 2\}^* \land \sum_{i \in \{0, 1, 2\}} x_i \equiv 0 \mod 3 \right\}$ is regular. That is, the set of all strings of 0’s, 1’s and 2’s such that the sum of the digits of the string is equal to 0 mod 3 is a regular language. For example, $1200201 \in L$, $00 \in L$, $\epsilon \in L$, $110010 \in L$, but $12100 \notin L$ and $2 \notin L$.

4. (5 points) Consider the following languages over the alphabet $\Sigma = \{0, 1, 2\}$:
   
   • $L_1 = \left\{ x_1...x_n \mid x_1...x_n \in \Sigma^* \land \sum_{i \in \{0, 1, 2\}} x_i \equiv 0 \mod 3 \right\}$
   
   • $L_2$ is the set of strings that contain a 1.
   
   Prove that $L_1 \cup L_2$ is regular. For example, $1200201 \in L_1 \cup L_2$, $00 \in L_1 \cup L_2$, $\epsilon \in L_1 \cup L_2$, $110010 \in L_1 \cup L_2$, $12100 \in L_1 \cup L_2$ but $2 \notin L_1 \cup L_2$. 
1. $a \{00, 11, 001, 111, 0001, 1101\}$
   
   $b \ 8$
   
   $c \{\varepsilon, 1, 01, 11, 101, 011, 111\}$
   
   $d \{\varepsilon, 1, 00, 01, 11, 001, 011, 101, 111, 0000, 0001, 0011, 0111, 1100, 1101, 1111, \}$

2. $L_1 \cap L_3 = \{0^{12k} \mid k \geq 0\}$, and it is infinite.

3. $L$ is accepted by the DFA $M = \{(q_0, q_1, q_2), \{0, 1, 2\}, \delta, q_0, \{q_0\}\}$ where state $q_i, 0 \leq i \leq 2$, "means" that the sum of the characters read so far sums to $i$ modulo 3.

   $$
   \begin{array}{c|ccc}
   \delta & 0 & 1 & 2 \\
   \hline
   q_0 & q_0 & q_1 & q_2 \\
   q_1 & q_1 & q_2 & q_0 \\
   q_2 & q_2 & q_0 & q_1 \\
   \end{array}
   $$

4. $L_1 \cup L_2$ is accepted by the DFA $M = \{(q_0, q_1, q_2, q_3), \{0, 1, 2\}, \delta, q_0, \{q_0, q_3\}\}$ where state $q_i, 0 \leq i \leq 2$, "means" that a 1 has not yet been read and the sum of the characters read so far sums to $i$ modulo 3, and state $q_3$ "means" that a 1 has been read.

   $$
   \begin{array}{c|cccc}
   \delta & 0 & 1 & 2 & \\
   \hline
   q_0 & q_0 & q_3 & q_2 \\
   q_1 & q_1 & q_3 & q_0 \\
   q_2 & q_2 & q_3 & q_1 \\
   q_3 & q_3 & q_3 & q_3 \\
   \end{array}
   $$