SEARCH TREE

**Node:** State in state tree

**Root node:** Top of state tree

**Children:** Nodes that can be reached from a given node in 1 step (1 operator)

**Expanding:** Generating the children of a node

**Open:** Node not yet expanded

**Closed:** Node after expansion

**Queue:** Ordered list of open nodes
SEARCH

BLIND SEARCH:  *Systematic Search*

**Depth–1st:** Continue along current path looking for goal

**Breadth–1st:** Expand all nodes at current level before progressing to next level

**Depth–limited Search:** Depth-1st + depth-limit

**Iterative Deepening Search:** limit=0, limit=1, ...

**USING COST:** \( g(n) = \text{cost from start to } n \)

**Branch-and-bound (= Uniform Cost Search):** Select node \( n \) with best \( g(n) \).

**USING HEURISTIC:** \( h(n) = \text{Estimate cost to a goal} \)

**Greedy Search:** Select node \( n \) with best \( h(n) \)

**A\(^*\):** Select node \( n \) with best \( f(n) = g(n) + h(n) \)

**IDA\(^*\):** A\(^*\) + \( f \)-cost limit.

**Hill-climbing:** Depth-1st exploring best \( h(n) \) first

**Simulated Annealing:** Hill climbing + random walk

**Beam Search:** Breadth-1st keeping only \( m \) nodes with best \( h(n)'s \) per level
DEPTH–1st SEARCH

1. Put start state onto queue

2. If queue is empty then fail

3. If head of queue is goal then succeed

4. Else remove head of queue, expand it, place children in front of queue

5. Recurse to 2
DEPTH–1st (cont.)

When to use

- Depth limited or known beforehand
- All solutions at same depth
- Any solution will do
- Possibly fast

When to avoid

- Large or infinite subtrees
- Prefer shallow solution
BREADTH-1st SEARCH

1. Put start state onto queue
2. If queue is empty then fail
3. If head of queue is goal then succeed
4. Else remove head of queue, expand it, place children at end of queue
5. Recurse to 2
BREADTH–1st (Cont.)

When to use

- Large or infinite search tree
- Solution depth unknown
- Prefer shallow solution

When to avoid

- Very wide trees
- Generally slow
- May need a lot of space
MODIFICATIONS TO DEPTH/BREADTH 1ST

Depth–limited Search:
Limit the total depth of the depth 1st search.

Iterative Deepening Search:
Repeat depth–limited search with limit 0, 1, 2, 3, … until a solution is found.

Bidirectional Search:
Simultaneously search forward from initial state and backward from goal state until both paths meet.
BRANCH–AND–BOUND
(= UNIFORM–COST SEARCH)

1. Put start state onto queue
2. If queue is empty then fail
3. If head of queue is goal then succeed
4. Else
   • remove head of queue,
   • expand it,
   • place in queue, and
   • sort entire queue with least cost-so-far nodes in front
5. Recurse to 2
BRANCH-AND-BOUND SUMMARY

Advantages

- Optimal (when costs are non-negative)
- Complete

Disadvantages

- Can be inefficient

When to use

- Desire best solution
- Keep track of cost so far

When to avoid

- May not work with negative costs
- May be overly conservative
- Any solution will do

Potential improvement

- Dynamic Programming
BRANCH-AND-BOUND + DYNAMIC PROG.

1. Put start state onto queue
2. If queue is empty then fail
3. If head of queue is goal then succeed
4. Else
   - remove head of queue,
   - expand it,
   - place in queue,
   - remove redundant paths:
     Paths that reach the same node as other paths but
     are more expensive, and
   - sort entire queue with least cost-so-far nodes in front
5. Recurse to 2
GREEDY SEARCH
(= Winston’s BEST–1st SEARCH)

1. Put start state onto queue
2. If queue is empty then fail
3. If head of queue is goal then succeed
4. Else
   • remove head of queue,
   • expand it,
   • place in queue, and
   • sort entire queue with least estimated-cost-to-goal nodes in front
5. Recurse to 2
GREEDY SEARCH SUMMARY

Advantages

• Can be very efficient
• Paths found are likely to be short

Disadvantages

• Neither optimal nor complete

When to use

• Desire ”short” solution

When to avoid

• When an optimal solution is required
A*

1. Put start state onto queue
2. If queue is empty then fail
3. If head of queue is goal then succeed
4. Else remove head of queue, expand it, place in queue, and sort entire queue with least cost-so-far + estimated-cost-remaining nodes in front
5. If multiple paths reach a common goal, keep only lowest cost-so-far path
6. Recurse to 2

- $f(\text{node}) = g(\text{node}) + h(\text{node})$, where
  - $f(\text{node}) = \text{estimated total cost}$
  - $g(\text{node}) = \text{cost-so-far to node}$
  - $h(\text{node}) = \text{estimated-cost-remaining (heuristic)}$.

- Properties of $h$:
  - Lower bound ($\leq$ actual cost)
  - Nonnegative
A* SUMMARY

Advantages

- Complete
- Optimal, when \( h \) is an underestimate
- Optimally efficient among all optimal search algorithms

Disadvantages

- Very high space complexity

When to use

- Desire best solution
- Keep track of cost so far
- Heuristic information available

When to avoid

- No good heuristics available
HILL CLIMBING SEARCH

1. Put start state onto queue
2. If queue is empty then fail
3. If head of queue is goal then succeed
4. Else remove head of queue, expand it, place children sorted by $h(n)$ in front of queue
5. Recurse to 2
HILL CLIMBING SUMMARY

Advantages

- Complete if backtracking is allowed (like in Winston’s book) and the graph is finite

Disadvantages

- Not optimal

When to use

- Depth limited or known beforehand
- All solutions at same depth
- Desire good solution
- Reliable estimate of remaining distance to goal
- Fast if good estimate

When to avoid

- If optimal solution is required
- Large or infinite subtrees
- No good estimate
- Difficult terrain
**BEAM SEARCH**

1. Put start state onto queue
2. If queue is empty then fail
3. If head of queue is goal then succeed
4. Else remove head of queue, expand it, place children at end of queue
5. If finishing a level, keep only $w$ best nodes in queue
6. Recurse to 2
BEAM SEARCH SUMMARY

Advantages

• Saves space

Disadvantages

• Neither optimal nor complete

When to use

• Large or infinite search tree
• Solution depth unknown
• Prefer shallow solution
• Possibly fast
• No more than \( wb \) nodes stored

When to avoid

• Can’t tell which solutions to prune
• Prefer conservative
SEARCH STRATEGIES -
Completeness; Optimality; and Time and Space Complexity

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<th>Optimal?</th>
<th>Time</th>
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<td>$ms$</td>
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(adapted from Russell & Norvig’s book)

- Y*: Yes, IF cost of a path is equal to its length. Otherwise No.
- b: branching factor
- s: depth of the solution
- d: maximum depth of the search tree
- l: depth limit
- m: beam size
- exp: exponential depending on heuristic $h$
- dep: depends on heuristic $h$
SEARCH STRATEGIES IN WINSTON’S BOOK
Summary

**Depth 1st:** Continue along current path looking for goal

**Breadth 1st:** Expand all nodes at current level before progressing to next level

**Hill Climbing:** Like depth 1st, but explore most promising children first

**Beam:** Like breadth 1st, but prune unpromising children

**Best 1st:** Expand best open node regardless of its depth

**Branch-and-bound:** Expand the least-cost-so-far node until goal reached

**A**: Like branch-and-bound, but with heuristic information