Name_____

QUIZ 5

Either prove or give a counterexample to establish whether or not each of the following CONJECTURES holds for every connected weighted graph $G = (V, E)$, $w : E \rightarrow \Re^+$, with $|E| > |V| \geq 3$, with distinct edge weights? You may use the fact that if the edge weights are distinct then G admits exactly one MST.

CONJECTURE 1: If e is the second lightest edge of E, then e belongs to every minimum spanning tree of G.

The CONJECTURE is true. Let uv be a lightest edge. Applying the Blue Rule in cut $(\{u\}, V - \{u\})$ colors uv Blue. Letting A be the set containing $\{u, v\}$ plus one endpoint of the second lightest edge, an application of the Blue Rule will color the second lightest edge Blue, assuring us that it belongs to an MST.

CONJECTURE 2: If e is the third lightest edge of E, then e belongs to every minimum spanning tree of G.

Edge uv in the following graph is the third lightest edge, but it does not belong to an MST.
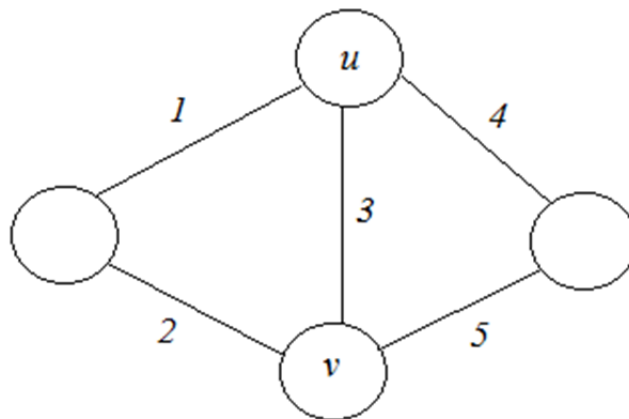
CONJECTURE 3: If e is the heaviest edge of E, then e does not belong to any minimum spanning tree of G.

Edge uv in the following graph is the heaviest edge, but it belongs to the MST.