Quiz 12 Name________________________________

Suppose that we are given a network $G=(V,E)$, source and sink vertices $\sigma,\tau \in V$, capacities $c: E \to \mathbb{Z}^+$, maximum flow *f* in *G*, and a pair of vertices $u,v \in V$ such that $uv \notin E$. We want to compute the maximum flow f^* in $G^* = (V, E \cup \{uv\})$ where the capacities of all edges in G^* are the same as the capacities in *G* except that *c*(*uv*)=6. Find an algorithm to compute f^* in time in $O(|V|+|E|)$.

<u>SOLUTION</u>: We add *uv* to E with 0 flow and capacity 6. We seek an augmenting path from σ to τ with respect to *f* in time in $O(|V|+|E|)$. If the increase in flow is less than 6, then other augmenting paths must be sought in the network with updated flows. Since each increase in flow is at least 1, then at most 6 augmentations can be found, yielding a time in $O(|V|+|E|)$.

The following graph, with all edges having capacity 1 and all edges directed from left to right, shows why you may have to find augmenting paths up to 6 times: