QUIZ 1

Suppose you are given an array *A*[1..*n*] of *n* distinct integers and some fixed *k*, $1 \le k \le n$. You should think of *k* as small and *n* as arbitrarily large. We seek the *k* largest elements of *A*. For example, if *n*=5 and *k*=2 and *A* is the array:

1	2	3	4	5
6	22	12	75	3

then the answer would be 75 and 22.

a Use the decision tree model (the basic operation is pairwise comparisons) to give a lower bound on the worst-case complexity of the problem.

b Give a linear (in *n*) time upper bound on the worst-case complexity of this problem.

SOLUTIONS: ***a*** Each subset of *k* elements of *A* is a possible answer, so the decision tree must have at least $\binom{n}{k}$ leaves. Fixing *k*, we note that $\binom{n}{k} = \frac{n(n-1)...(n-k+1)}{k!} \in \Omega(n^k)$. So the height of the tree must be in $\Omega(\lg n^k) = \Omega(k \lg n)$, which is $\Omega(\lg n)$ for fixed *k*. Note that the bound is not tight.

b We find *A*'s largest element, and return it and remove it, *k* times.

for $i \leftarrow 1$ **to** *k* **do**

    MAXINDEX $\leftarrow 1$

    for $j \leftarrow 2$ **to** *n*-1 **do**

        if *A*[**MAXINDEX**]<*A*[*j*] **then MAXINDEX** $\leftarrow j$

    return *A*[MAXINDEX]

    ***A*[MAXINDEX]** $\leftarrow -\infty$