CS524
HW#6

DUE: Monday, December 12

1. (8 points) Suppose that we are given

- network $G=(V,E)$,
- $\sigma,\tau \in V$,
- $c: E \to \Re^+$,
- maximum flow *f* in *G*,
- edge $e \in E$.

We want to compute the maximum flow f^* in $G^*=(V,E)$, $\sigma,\tau \in V$, $c^*: E \to \Re^+$ where

$$c^*(\varepsilon)=\begin{cases} c(\varepsilon), \text{if } \varepsilon \neq e \\ c(\varepsilon)+1, \text{if } \varepsilon = e \end{cases}.$$

That is, G^* is identical to *G* except that the capacity of *e* is increased by 1. Find an algorithm to compute f^* in time in $O(|V|+|E|)$.

2. (5 points) Prove or give a counterexample to the following.

<u>Conjecture</u>: Assume that for arbitrary network $G=(V,E), \sigma,\tau \in V$, with capacities $c: E \to \Re^+$, cut (S,T) is a minimum cut. In network $G^*=(V,E), \sigma,\tau \in V$, with capacities $c^*: E \to \Re^+$ with $c^*(e)=c(e)+4$ for all $e \in E$, cut (S,T) must be a minimum cut.

3. (3 points) Do **Exercise *34.2-1*** on page 982 of our text.

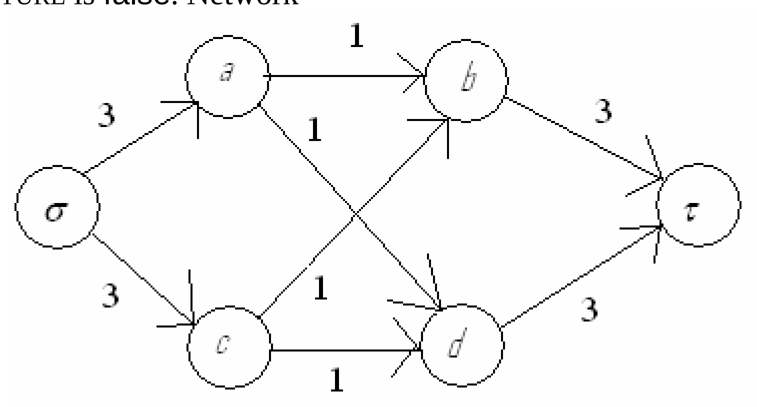
4. (3 points) Do **Exercise *34.5-7*** on page 1017 of our text.

C.S.524

SOLUTION FOR H.W. #6

1. We form a graph $\Upsilon(G^*)$ with edges of positive residual capacity relative to *f*. That is, $\Upsilon(G^*) = (V, E')$ where $E' = \{\varepsilon \in E \mid c^*(\varepsilon) - f(\varepsilon) > 0\}$. Each $\varepsilon \in E'$ has capacity $\kappa(\varepsilon) = c^*(\varepsilon) - f(\varepsilon)$. That is, the capacity of ε is its residual capacity relative to *f*. Constructing $\Upsilon(G^*)$ takes time in $O(|V|+|E|)$. We seek a path in $O(|V|+|E|)$ from σ to τ. If such a path exists, it is an augmenting path relative to *f*, and it can be found in time in $O(|V|+|E|)$.

2. The CONJECTURE is false. Network



has minimum cut $(\{\sigma, a, c\}, \{b, d, \tau\})$. But the network obtained when 4 is added to every capacity has two min cuts, $(\{\sigma\}, \{a, b, c, d, \tau\})$ and $(\{\sigma, a, b, c, d\}, \{\tau\})$.

3. A certificate for isomorphism of $G = (V, E)$ and $G' = (V', E')$ is a bijection $f : V \to V'$. The fact that *f* preserves adjacency can be verified be checking adjacency of the $\binom{|V|}{2} \in O(|V|^2)$ pairs of vertices. That is, **for each** $u, v \in V$ we verify that $(u, v) \in E$ if and only if $(f(u), f(v)) \in E'$.

4. We know that the HAMILTON CYCLE problem is NP-complete. It is easy to see that

the HAMILTON-CYCLE problem $\leq_P$ the LONGEST-SIMPLE-CYCLE problem because graph $G=(V,E)$ admits a Hamilton cycle if and only if the length of its longest simple cycle equals $|V|$.