CS524
HW#6

DUE: Monday, April 18

1. (10 points) A herpetologist has collected *n* salamanders, which are believed to belong to two different species. She examines *m* randomly drawn pairs, $m \le \binom{n}{2}$, and labels the examined pairs as either *same* or *different*, depending upon whether she thinks they're the same species or different. We want to test whether her decisions are consistent. For example, if she judged that:

- *a* and *b* are the *same*,
- *a* and *c* are the *same*,
- *b* and *c* are *different*,

then her decisions are inconsistent. Show how to test, in $O(n+m)$ time, whether or not her decisions are consistent.

2. (12 points) Given a graph $G = (V, E)$, a set of vertices $U \subseteq V$ is an *independent set* if no pair of vertices of *U* has an edge between them. Consider the INDEPENDENT SET problem:
INSTANCE: Graph $G = (V, E)$ and $k \in \mathbb{Z}^+$.
QUESTION: Does *G* have an independent set of cardinality *k*?

a Prove that the INDEPENDENT SET problem is NP-complete.
Hint: You may want to use the VERTEX-COVER problem described in **Section 34.5.2** of our text.

b Suppose you have an $\mathfrak{Oracle}$ which will decide on the INDEPENDENT SET problem in constant time. That is, the $\mathfrak{Oracle}$ will accept as input a graph $G = (V, E)$ and $k \in \mathbb{Z}^+$ and will tell, in constant time, whether or not *G* has an independent set of cardinality *k*. Show how to use the $\mathfrak{Oracle}$ to determine the largest independent set of *G* in polynomial (in |*V*| and |*E*|) time. Note that we seek an answer to the optimization problem using a solution to the decision problem (the $\mathfrak{Oracle}$). Analyze your algorithm.