1. (10 points) Describe a $O(n^2)$ dynamic programming algorithm to find the length of the longest (not necessarily contiguous) increasing sequence of integers of $A[1..n]$. For example, if $A = (11,17,5,8,6,4,7,7,12,3)$, then the answer would be 4 because of the subsequence $(5,6,7,12)$.

2. (10 points) Do Problem 15-6 on pg. 368 of our text. Your algorithm should work in time in $\Theta(n^2)$. 


1. For every $i$, we compute $\text{Length\_of\_longest}[i]$, the length of the longest increasing subsequence in $A[1..i]$ whose rightmost member is $L[i]$. The dynamic programming formulation is

$$
\text{Length\_of\_longest}[i] = \max_{1 \leq j < i} \{1, \text{Length\_of\_longest}[j] + 1\}
$$

This translates to the program

```plaintext
for $i \leftarrow 2$ to $n$ do
    $\text{Length\_of\_longest}[i] \leftarrow 1$
    for $j \leftarrow 1$ to $i-1$ do
        if $A[j] < A[i]$ and $\text{Length\_of\_longest}[j] + 1 > \text{Length\_of\_longest}[i]$ then
            $\text{Length\_of\_longest}[i] \leftarrow \text{Length\_of\_longest}[j] + 1$
return $\max_{1 \leq i \leq n} \{\text{Length\_of\_longest}[i]\}$
```

2. We compute $d[i,j]$, $1 \leq i, j \leq n$, to be the most profitable way to get from the bottom row to square $(i,j)$. For $1 < i \leq n, 1 \leq j \leq n$, we note that we move to square $(i,j)$ from one of the two or three squares below it, and the dynamic programming formulation is

$$
d[i,j] = \max \begin{cases}
    d[i-1,j-1] + p((i-1,j-1),(i,j)), & \text{if } j > 1 \\
    d[i-1,j] + p((i-1,j),(i,j)), & \\
    d[i-1,j+1] + p((i-1,j+1),(i,j)), & \text{if } j < n
\end{cases}
$$

with the initialization $d[1,j]=0$ for $1 \leq j \leq n$.

Finally, the answer to the question is $\max_{1 \leq i \leq n} \{d[n,j]\}$

```plaintext
for $j \leftarrow 1$ to $n$ do $d[1,j] \leftarrow 0$
for $i \leftarrow 2$ to $n$ do
    for $j \leftarrow 1$ to $n$ do
        $d[i,j] = \max \begin{cases}
            d[i-1,j-1] + p((i-1,j-1),(i,j)), & \text{if } j > 1 \\
            d[i-1,j] + p((i-1,j),(i,j)), & \\
            d[i-1,j+1] + p((i-1,j+1),(i,j)), & \text{if } j < n
        \end{cases}$
return $\max_{1 \leq i \leq n} \{d[n,j]\}$
```