CS524
HW#5

DUE: Wednesday, December 1

1. (10 points) Describe a $O(n^2)$ dynamic programming algorithm to find the length of the longest (not necessarily contiguous) increasing sequence of integers of $A[1..n]$. For example, if $A = (11, 17, 5, 8, 6, 4, 7, 7, 12, 3)$, then the answer would be 4 because of the subsequence $(5, 6, 7, 12)$.

2. (10 points) Do **Problem *15-6*** on pg. 368 of our text. Your algorithm should work in time in $\Theta(n^2)$.

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HW#5 SOLUTIONS

1. For every i, we compute *Length_of_longest*[i], the length of the longest increasing subsequence in A[1..i] whose rightmost member is L[i]. The dynamic programming formulation is

$$Length\_of\_longest[i] = \max_{\substack{1 \le j < i \\ A[j] < A[i]}} \{1, Length\_of\_longest[j] + 1\}$$

This translates to the program

$Length\_of\_longest[1] \leftarrow 1$
for $i \leftarrow 2$ **to** n **do**
  $Length\_of\_longest[i] \leftarrow 1$
  for $j \leftarrow 1$ **to** i-1 **do**
    if $A[j] < A[i]$ **and** *Length_of_longest*[j]+1> *Length_of_longest*[i]
    then *Length_of_longest*[i]$\leftarrow$ *Length_of_longest*[j]+1
return $\max_{1 \le i \le n} \{Length\_of\_longest[i]\}$

2. We compute $d[i,j]$, $1 \le i, j \le n$, to be the most profitable way to get from the bottom row to square (i,j). For $1 < i \le n, 1 \le j \le n$, we note that we move to square (i,j) from one of the two or three squares below it, and the dynamic programming formulation is

$$d[i,j] = \max \begin{Bmatrix} d[i-1, j-1] + p((i-1, j-1), (i, j)), \text{ if } j > 1 \\ d[i-1, j] + p((i-1, j), (i, j)), \\ d[i-1, j+1] + p((i-1, j+1), (i, j)), \text{ if } j < n \end{Bmatrix}$$

with the initialization $d[1,j]=0$ for $1 \le j \le n$.

Finally, the answer to the question is $\max_{1 \le j \le n} \{d[n, j]\}$

for $j \leftarrow 1$ **to** n **do** $d[1, j] \leftarrow 0$
for $i \leftarrow 2$ **to** n **do**
  for $j \leftarrow 1$ **to** n **do**

$$d[i,j] = \max \begin{Bmatrix} d[i-1, j-1] + p((i-1, j-1), (i, j)), \text{ if } j > 1 \\ d[i-1, j] + p((i-1, j), (i, j)), \\ d[i-1, j+1] + p((i-1, j+1), (i, j)), \text{ if } j < n \end{Bmatrix}$$

return $\max_{1 \le j \le n} \{d[n, j]\}$