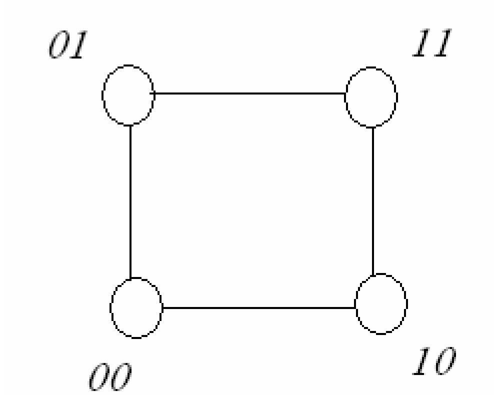
# CS524
# HW#4

**DUE**: Monday, March 21

1. (12 points) The *n-dimensional cube*, $Q_n$, has $2^n$ vertices consisting of all *n*-bit numbers, and two vertices are adjacent (there is an edge between them) if and only if the corresponding numbers differ in exactly one position. Thus, in $Q_5$, vertices 01000 and 00010 are not adjacent, but vertices 01000 and 01010 are adjacent. And $Q_2$ is




An *n-bit Gray code* is a Hamiltonian cycle in $Q_n$, that is a sequence of vertices $v_1,\ldots,v_{2^n}$ such that $v_i$ is adjacent to $v_{i+1}$ for $1 \le i < n$ and $v_1$ is adjacent to $v_{2^n}$. Let $g(n)$ denote the number of *n*-bit Gray codes. The 2-bit Gray codes, $(00, 01, 11, 10)$ and $(00, 10, 11, 01)$ show that $g(2)=2$. Sloane's *Online Encyclopedia of Integer Sequences* shows the following small values of *g*:

$$g(3)=12,\ g(4)=2688 \text{ and } g(5)=1813091520$$

Design a program to estimate $g(n)$, the number of *n*-bit Gray codes.

2. (12 points) Do **PROBLEM *5-2***, parts ***a***, ***b***, ***c*** and ***d***, on pages 118→119 of our text.

# CS524
## HW#4 SOLUTIONS

2. ***a***

```
NumberIndices ← 0                              ▸ 0 values of A have been probed
for i ← 1 to n do
    B[i] ← 0                                   ▸ A[i] hasn't been probed
 i ← RANDOM(1,n)
 if A[i]=x then return i
          else do
               if B[i]=0 then NumberIndices ← NumberIndices + 1
                B[i] ← 1
                if NumberIndices=n then return "x not in A"
```

***b*** Call a trial success if $i$ is chosen. The probability of success is $1/n$.Since each trial is independent of all previous trials, the rv which denotes the number of trials until the first success is geometrically distributed, and the expected value of the rv is $\frac{1}{1/n} = n$.

***c*** This is the same as part ***b*** except that the probability of success is $k/n$. Thus, the expected number of trials is $\frac{1}{k/n} = \frac{n}{k}$.

***d*** Since every trial results in failure, this is equivalent to the COUPON COLLECTOR'S PROBLEM, which has an expected number of trials $nH_n$.