CS3133

MIDTERM EXAM

Name_____________

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All documentation permitted

1. (25 points) Prove or give a counterexample to the following:
CONJECTURE: For any nonempty regular language *L*, if the shortest string in *L* is of length *n*, then **any** DFA to accept *L* must have at least *n*+1 states.

2. (25 points) For any languages L_0, L_1, we define

$$L_0 \odot L_1 = \{z \mid (z \in L_0 \wedge z \notin L_1) \vee (z \notin L_0 \wedge z \in L_1)\}.$$

So, for example, $\{\varepsilon, 011, 1, 01\} \odot \{0, 011, 000, 01\} = \{\varepsilon, 1, 0, 000\}$. Prove or give a counterexample to the following:

CONJECTURE: The set of regular languages is closed under the operation $\odot$.

3. (20 points) Prove or give a counterexample to the following:

<u>CONJECTURE</u>: For **any** NFAs $N = (Q, \Sigma, \Delta, S, F)$ and $N' = (Q, \Sigma, \Delta, S, Q - F)$, it must be the case that $L(N') = \sim L(N)$, where $Q - F = \{q \mid (q \in Q) \wedge (q \notin F)\}$ and $\sim L(N) = \Sigma^* - L(N)$.

4. (30 points) ***a*** Is $\left\{0^n 1^{2n} \mid n \geq 0\right\}$ regular? Justify your answer.

b Is $\left\{0^n 1^m \mid n, m \geq 0\right\}$ regular? Justify your answer.

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Solutions to Midterm Exam

1. The CONJECTURE is true. Let $Q^* \subseteq Q$ be the set of states reachable from *s* in any $M = (Q, \Sigma, \delta, s, F)$ such that $L = L(M)$. Because *L* is nonempty, Q^* is nonempty, and $s \in Q^*$ and $Q^* \cap F \neq \varnothing$. If there were a state $q \in Q^* \cap F$ accessible on a path from *s* at a distance less than *n* from *s*, then this path would correspond to a word from *L* of length less than *n*. But since such a word does not exist, there must be a $q \in Q^* \cap F$ at a distance at least *n* from *s*. So Q^*, and hence *Q*, must contain at least *n*+1 states.

2. Regular sets are closed under set difference, $A - B = \{x | (x \in A) \wedge (x \notin B)\}$, since $A - B = A \cap \sim B$ and regular sets are closed under intersection and complement. So they are also closed under $\odot$ since $L_0 \odot L_1 = (L_0 - L_1) \cup (L_1 - L_0)$.

3. The CONJECTURE is false. NFA $N = (\{q\}, \{0,1\}, \Delta, \{q\}, \{q\})$ with $\Delta(q,0) = \{q\}$ accepts 0^*. But $N' = (\{q\}, \{0,1\}, \Delta, \{q\}, \varnothing)$ accepts $\varnothing$. And string 1 doesn't belong to either $L(N)$ or $L(N')$.

4. ***a*** $\{0^n 1^{2n} \mid n \geq 0\}$ is not regular. Assume it is regular, and let *k* be the integer guaranteed by the Pumping Lemma for Regular Languages. Choosing $z = 0^k 1^{2k}$, we know that *z* can be written as $z = uvw$ with $|uv| \leq k$ and $|v| \geq 1$. So *v* is a nonempty string of 0's. By the Pumping Lemma, $uv^2 w \in \{0^n 1^{2n} \mid n \geq 0\}$. But $uv^2 w$ does **not** have twice as many 1's as 0's, which is a contradiction. So $\{0^n 1^{2n} \mid n \geq 0\}$ can not be regular.

b $\{0^n 1^m \mid n, m \geq 0\}$ is regular because it is accepted by the DFA $M = (\{q_0, q_1, q_{blackhole}\}, \{0,1\}, \delta, q_0, \{q_0, q_1\})$ where

δ	0	1
q_0	q_0	q_1
q_1	$q_{blackhole}$	q_1
$q_{blackhole}$	$q_{blackhole}$	$q_{blackhole}$