CS3133

MIDTERM EXAM

Name______________

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All documentation permitted

1. (20 points) For each of the following languages over {0,1}, what is a shortest string that does **not** belong to the language? That is, for each L describe w such that $w \notin L$ and if $|z| < |w|$ then $z \in L$. For example, if $L = 101 + 00$, then the answer would be ε.

a 0^*1^*

b $0(10)^*1$

c $0^* + 1^*$

d $(\varepsilon + 0)1$

e $(\varepsilon + 0 + 10 + 11)(0+1)^*$

2. (20 points) Consider the following sublanguage L of C.

L is the set of all binary strings preceded by /# and followed by #/

So, for example, $/\#001101\#/ \in L$ and $/\#\#/ \in L$, but $/\#\#0/ \notin L$ and $/\#/\#\#/\#/ \notin L$ and $/\#\#//\#01\#/ \notin L$.

Is L regular? Justify your answer.

3. (30 points) Is $\left\{0^m 1^n 0^{m+n} \mid m \geq 0, n \geq 0\right\}$ regular? Justify your answer.

4. (30 points) Tell whether or not each of the following languages over {0, 1} is regular. Justify your answers

a The set of strings that don't contain either an 0101 or a 11010.

b All strings with at least two occurrences of the substring 11. Note that 111 has two occurrences of 11.

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Solutions to Midterm Exam

1. ***a*** 10

b ε

c 01 or 10

d ε

e Since $L\left(\left(\varepsilon+0+10+11\right)\left(0+1\right)^*\right)=\left(0+1\right)^*$, every binary string belongs to the language and there does not exist a shortest string which doesn't belong.

2. *L* is regular over $\Sigma=\{0,1,/,\#\}$ because it is described by the regular expression

$$/\#\left(0+1\right)^*\#/ \quad .$$

3. *L* is not regular. Assume *L* is regular, and let *k* be as guaranteed by the Pumping Lemma. Then $0^{k+1}10^{k+2}\in L$ can be written as *uvw* with $0<|uv|\le k$ and $|v|\ge 1$. So *v* must be a nonempty string of 0's, and the number of 0's in uv^3w is larger than the number of 0's following the 1's in uv^3w. So $uv^3w\notin L$. But by the Pumping Lemma for regular languages, uv^3w must belong to *L*. By this contradiction, *L* can not be regular.

4. ***a*** The set of binary strings that contain 0101 or 11010 (at least one of them) is described by

$$\left(0+1\right)^*0101\left(0+1\right)^*+\left(0+1\right)^*11010\left(0+1\right)^*,$$

so it is regular. Since the set of regular languages is closed under complement (the complement of any regular language is regular), then the set of strings that don't contain either an 0101 or a 11010 must be regular.

b Every string in the language includes a 111 or two disjoint 11's, or both.

$$\left(0+1\right)^*111\left(0+1\right)^*+\left(0+1\right)^*11\left(0+1\right)^*11\left(0+1\right)^*$$