CS3133

MIDTERM EXAM

Name______________

Date: September 17, 2007

All documentation permitted

1. (25 points) Construct a **D**FA to accept $(aa+bb+cc)^*$. Note that it must be deterministic (not an NFA).

2. (25 points) Prove or give a counterexample to the following. Make sure it is clear whether you believe the CONJECTURE is true or false.

CONJECTURE: For any finite nonempty alphabet Σ and any regular language L over Σ, the language $L_1 = \{wyz \mid w \in L \wedge y, z \in \Sigma\}$ must be regular.

As an example, if $\Sigma = \{0,1\}$ and $010 \in L$, then 01000, 01001, 01010 and 01011 must all belong to L_1.

3. (30 points) Is $\left\{0^m 1^n 0^{m+n} \mid m \geq 0, n \geq 0\right\}$ regular? Make sure it is clear whether you believe it is regular. Justify your answer.

4. (20 points) Find regular expressions for the following languages over {0, 1}.
a All strings with an even number of 0's.

b All strings with at least two occurrences of the substring 11. Note that 111 has two occurrences of 11.

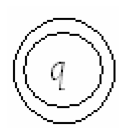
<u>**CS3133**</u>
Solutions to <u>Midterm Exam</u>

1. $\left(\{q_0, q_a, q_b, q_c, q_{blackhole}\}, \{a,b,c\}, \delta, q_0, \{q_0\}\right)$ with

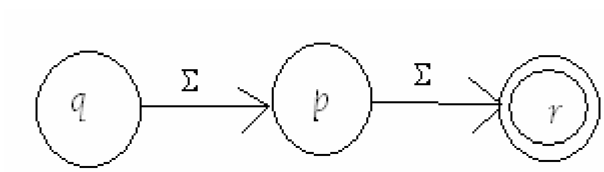
δ	a	b	c
q_0	q_a	q_b	q_c
q_a	q_0	$q_{blackhole}$	$q_{blackhole}$
q_b	$q_{blackhole}$	q_0	$q_{blackhole}$
q_c	$q_{blackhole}$	$q_{blackhole}$	q_0
$q_{blackhole}$	$q_{blackhole}$	$q_{blackhole}$	$q_{blackhole}$

2. The CONJECTURE is true. One way to see it is that if we know that L is regular, and clearly Σ, viewed as a language, is regular. Then because regular languages are closed are concatenation, then $L\Sigma$ must be regular, and so must $(L\Sigma)\Sigma = L_1$.

Another way is to see that if L is regular, then there exists an NFA $N = (Q, \Sigma, \Delta, S, F)$ such that $L(N)=L$. We construct a new NFA, N^*, by choosing 2 new states, p and r, not in Q, and adding the path



$\Downarrow$



from each $q \in F$. That is, $N^* = \left(Q \cup \{p, r\}, \Sigma, \Delta^*, S, \{r\}\right)$ where for all $q \in Q, a \in \Sigma$,

$$\Delta^*(q,a) = \begin{cases} \Delta(q,a), \text{if } q \in Q - F \\ \Delta(q,a) \cup \{p\}, \text{if } q \in F \end{cases}$$

and for all $a \in \Sigma$, $\Delta^*(p,a) = \{r\}$ and $\Delta^*(r,a) = \varnothing$.

3. $\left\{0^m 1^n 0^{m+n} \mid m \geq 0, n \geq 0\right\}$ is not regular. Assume L is regular, and let k be as guaranteed by the Pumping Lemma. Then $0^k 1^k 0^{2k}$ can be written as uvw with $|uv| \leq k$ and $|v| \geq 1$. So v must be a nonempty string of 0's, and the number of initial 0's and 1's in uv^2w is larger

than the number of 0's in 0^{2k}. So $uv^2w \notin L$. But by the Pumping Lemma for regular languages, uv^2w must belong to L. By this contradiction, L can not be regular.

4. ***a*** Every string can be decomposed into substrings which each have two 0's, or it is a string of 1's.

$$\left(1^*01^*01^*\right)^* + 1^*$$

b Every string includes a 111 or two disjoint 11's.

$$(0+1)^*111(0+1)^* + (0+1)^*11(0+1)^*11(0+1)^*$$