1. (25 points) Is the set of all binary strings which do not contain a 11 a regular set? Justify your response. For example, $\varepsilon$ and 01010 belong to the language, but 011 does not.
2. (25 points) Describe an NFA (you may use $\varepsilon$-transitions) to accept the language 

$$(0+1)^*1(0+1)^* + 0$$.
3. (25 points) Give a context-free grammar to generate the language
\[ \{ 0^n1^n \mid m, n \geq 0 \land m \neq n \} \]. (Hint: \( m \neq n \) if and only if \( m > n \) or \( m < n \)).
4. (25 points) Is \( L = \left\{ a^m b^n c^n \mid n \geq 0 \right\} \) regular? Justify your response.
CS3133
Solutions to Midterm Exam

1. It is easy to see that the set of all binary strings which do contain a 11 is regular. It is accepted by DFA $M = (\{s, p, q\}, \{0,1\}, \delta, s, \{q\})$ where

$$\delta \quad \begin{array}{l}
0 \quad 1 \\
\quad s \\
\quad p \\
\quad s \\
\quad q \\
\quad q \\
\end{array}$$

The answer follows from the fact that regular languages are closed under complement. Also, one could accept the language with the DFA $M^* = (\{s, p, q\}, \{0,1\}, \delta, s, \{s, p\})$.

2. An NFA to accept $(0+1)^*1(0+1)^*$ is $M_0 = (\{q_0, q_1, q_2\}, \{0,1\}, \Delta_0, \{q_0\}, \{q_1\})$, where

$$\Delta_0 \quad \begin{array}{l}
0 \quad 1 \\
q_0 \quad \{q_0\} \\
q_1 \quad \{q_1\} \\
q_2 \quad \emptyset \\
\end{array}$$

An NFA to accept 0 is $M_1 = (\{p_0, p_1\}, \{0,1\}, \Delta_1, \{p_0\}, \{p_1\})$, where

$$\Delta_1 \quad \begin{array}{l}
0 \quad 1 \\
p_0 \quad \{p_1\} \\
p_1 \quad \emptyset \\
\end{array}$$

An NFA to accept $(0+1)^*1(0+1)^*+0$ is obtained by using $M_0$ and $M_1$, and adding an $\varepsilon$-transition to the initial state of each machine. That is, $M = (\{q_0, q_1, q_2, p_0, p_1, s\}, \{0,1\}, \Delta, \{q_1, p_1\})$, where $\Delta$ acts like $\Delta_0$ and $\Delta_1$ on the respective states of those machines, and

$$\Delta \quad \begin{array}{l}
\varepsilon \quad 0 \quad 1 \\
\quad s \\
\quad \{q_0, p_0\} \\
\quad \emptyset \\
\quad \emptyset \\
\end{array}$$

3. We generate $\{0^n1^n | m > n \geq 0\} \cup \{0^n1^n | n > m \geq 0\}$. The first language is generated from $S_0$ and the second from $S_1$. From $S_2$ we generate $0^n1^n$. 

4. L is not regular. Assume L is regular, and let k be as guaranteed by the Pumping Lemma. Then $a^kb^kc^k$ can be written as $uvw$ with $|uv| \leq k$ and $|v| \geq 1$. So v is a nonempty string of a's, and $uv^2w$ contains more a's than b's or c's. So $uv^2w \notin L$. But by the Pumping Lemma for regular languages, $uv^2w$ must belong to L. By this contradiction, L can not be regular.