CS3133

MIDTERM EXAM

Name______________

Date: September 19, 2005

All documentation permitted

1. (25 points) Is the set of all binary strings which do **not** contain a 11 a regular set? Justify your response. For example, ε and 01010 belong to the language, but 011 does not.

2. (25 points) Describe an NFA (you may use ε-transitions) to accept the language

$$(0+1)^{*}1(0+11)^{*}+0 \quad .$$

3. (25 points) Give a context-free grammar to generate the language $\left\{0^m 1^n \mid m, n \geq 0 \wedge m \neq n\right\}$. (Hint: $m \neq n$ if and only if $m > n$ or $m < n$).

4. (25 points) Is $L = \left\{ a^n b^n c^n \mid n \geq 0 \right\}$ regular? Justify your response.

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Solutions to Midterm Exam

1. It is easy to see that the set of all binary strings which do contain a 11 is regular. It is accepted by DFA $M = (\{s, p, q\}, \{0,1\}, \delta, s, \{q\})$ where

δ	0	1
s	s	p
p	s	q
q	q	q

The answer follows from the fact that regular languages are closed under complement. Also, one could accept the language with the DFA $M^* = (\{s, p, q\}, \{0,1\}, \delta, s, \{s, p\})$.

2. An NFA to accept $(0+1)^* 1(0+11)^*$ is $M_0 = (\{q_0, q_1, q_2\}, \{0,1\}, \Delta_0, \{q_0\}, \{q_1\})$, where

Δ_0	0	1
q_0	$\{q_0\}$	$\{q_0, q_1\}$
q_1	$\{q_1\}$	$\{q_2\}$
q_2	$\varnothing$	$\{q_1\}$

An NFA to accept 0 is $M_1 = (\{p_0, p_1\}, \{0,1\}, \Delta_1, \{p_0\}, \{p_1\})$, where

Δ_1	0	1
p_0	$\{p_1\}$	$\varnothing$
p_1	$\varnothing$	$\varnothing$

An NFA to accept $(0+1)^* 1(0+11)^* + 0$ is obtained by using M_0 and M_1, and adding an ε-transition to the intial state of each machine. That is,

$$M = (\{q_0, q_1, q_2, p_0, p_1, s\}, \{0,1\}, \Delta, \{s\}, \{q_1, p_1\})$$

where Δ acts like Δ_0 and Δ_1 on the respective states of those machines, and

Δ	ε	0	1
s	$\{q_0, p_0\}$	$\varnothing$	$\varnothing$

3. We generate $\{0^m 1^n \mid m > n \geq 0\} \cup \{0^m 1^n \mid n > m \geq 0\}$. The first language is generated from S_0 and the second from S_1. From S_2 we generate $0^n 1^n$.

$$S \to S_0 \mid S_1$$
$$S_0 \to AS_2$$
$$A \to 0 \mid A0$$
$$S_2 \to 0S_21 \mid \varepsilon$$
$$S_1 \to S_2B$$
$$B \to 1 \mid B1$$

4. *L* is not regular. Assume *L* is regular, and let *k* be as guaranteed by the Pumping Lemma. Then $a^kb^kc^k$ can be written as *uvw* with $|uv| \leq k$ and $|v| \geq 1$. So *v* is a nonempty string of *a*'s, and uv^2w contains more *a*'s than *b*'s or *c*s. So $uv^2w \notin L$. But by the Pumping Lemma for regular languages, uv^2w must belong to *L*. By this contrdiction, *L* can not be regular.