1. (20 points) Is $L = \{ w \in \{0,1\}^* \mid |w| \text{ is divisible by } 3 \}$ regular? Justify your response.
2. (30 points) For each of $L_0$ and $L_1$, tell whether or not the language is regular. Justify your responses.

$L_0 = \text{the set of binary strings that contain at least two occurrences of 01}$

$L_1 = \text{the set of binary strings that contain fewer than two occurrences of 01}$. 
3. (25 points) Consider the language \( L \) consisting of the set of strings of balanced parentheses over \( \Sigma = \{ (, ) \} \). That is, a string \( w \) belongs to \( L \) if \( w \) contains just as many (‘s as )’s, and there is a pairing between the (‘s and the )’s such that each ( is paired with a ) to its right in \( w \). For example, \( (()) \in L, \in L, ((())) \in L, () \in L, () \in L \) but \( ) (\in L \) and \( (()) \notin L \). Is \( L \) regular? Justify your response.
(25 points) Design a context-free grammar for the language

\[ L = \{0^n1^n0^{2m} | n \geq 0 \land m \geq 0\} = \{0^n1^n0^{2m} | n \geq 0 \land m \geq 0\} \]

For example, 0011000 \in L but 001110 \not\in L.
1. \( L_0 = \left( (0+1)^* (0+1)^* (0+1)^* \right)^* \), so it is regular.

2. Both languages are regular. \( L_0 = (0+1)^* 0^1 (0+1)^* 0^1 (0+1)^* \) and because
   \( L_1 = \overline{L_0} = \{0,1\}^* - L_0 \) and regular languages are closed under complement, it follows that \( L_1 \) must be regular.

3. \( L \) is not regular. Assume \( L \) is regular, and let \( n \) be the integer provided by the Pumping Lemma. Let \( w = (\cdot)^n \). That is \( w \) is a string of \( n \) (\('\)\)'s followed by \( n \) (\('\)\)'s. \( w \) can be written \( xyz \) such that \( |xy| \leq n \) and \( |y| \geq 1 \). Thus, \( xy \) only spans (\('\)'s and \( y \) is a nonempty string of (\('\)'s. The Pumping Lemma forces \( \{ xy^k z | k \geq 0 \} \subseteq L \). But this is a contradiction since \( k \neq 1 \Rightarrow xy^k z \) does not have as many (\('\)\)'s as \( y \)\)'s. Thus, \( L \) is not regular.

4. 

\[
\begin{align*}
S & \rightarrow AB \\
A & \rightarrow 0A1|\varepsilon \\
B & \rightarrow 1B00|\varepsilon
\end{align*}
\]