CS3133

Midterm Exam

**Name**_____________

**<u>Date</u>**: September 13, 2002

**All documentation permitted**

1. (20 points) Is $L=\left\{w \mid w\in\{0,1\}^* \wedge |w| \text{ is divisible by } 3\right\}$ regular? Justify your response.

2. (30 points) For each of $L_0$ and $L_1$, tell whether or not the language is regular. Justify your responses.

$L_0$ = the set of binary strings that contain at least two occurences of 01

$L_1$ = the set of binary strings that contain fewer than two occurences of 01.

3. (25 points) Consider the language $L$ consisting of the set of strings of balanced parentheses over $\Sigma = \{(,)\}$ . That is, a string $w$ belongs to $L$ if $w$ contains just as many ('s as )'s, and there is a pairing between the ('s and the )'s such that each ( is paired with a ) to its right in $w$. For example, $(()()) \in L$ , $e \in L$ , $((())) \in L$ , $() \in L$ , $()() \in L$ but $)( \notin L$ and $()) \notin L$ . Is $L$ regular? Justify your response.

4 (25 points) Design a context-free grammar for the language

$$L = \left\{ 0^n 1^n 1^m 0^{2m} \mid n \geq 0 \wedge m \geq 0 \right\} = \left\{ 0^n 1^{n+m} 0^{2m} \mid n \geq 0 \wedge m \geq 0 \right\}.$$

For example, $0011100 \in L$ but $001110 \notin L$.

# **CS3133**

Solutions to Midterm Exam

1. $L_0 = \left((0+1)(0+1)(0+1)\right)^*$, so it is regular.

2. Both languages are regular. $L_0 = (0+1)^* 01(0+1)^* 01(0+1)^*$ and because $L_1 = \overline{L_0} = \{0,1\}^* - L_0$ and regular languages are closed under complement, it follows that $L_1$ must be regular.

3. $L$ is not regular. Assume $L$ is regular, and let $n$ be the integer provided by the Pumping Lemma. Let $w = (^n)^n$. That is $w$ is a string of $n$ ('s followed by $n$ )'s. $w$ can be written $xyz$ such that $|xy| \leq n$ and $|y| \geq 1$. Thus, $xy$ only spans ('s and $y$ is a nonempty string of ('s. The Pumping Lemma forces $\{xy^k z \mid k \geq 0\} \subseteq L$. But this is a contradiction since $k \neq 1 \Rightarrow xy^k z$ does not have as many ('s as )'s. Thus, $L$ is not regular.

4.

$$S \to AB$$
$$A \to 0A1 \mid e$$
$$B \to 1B00 \mid e$$