CS3133

Midterm Exam

Name______________

Date: September 25,2000

All documentation permitted

1. (12 points) Tell whether each of the following is true or false for all regular expressions *v* and *w*. For each identity you believe to be false, give examples for *v* and *w* for which it is false.

$a)\quad \left(v^{*}w^{*}\right)^{*} = \left(v \cup w\right)^{*}$

$b)\quad w\left(vw \cup w\right)^{*} v = vv^{*}w\left(vv^{*}w\right)^{*}$

$c)\quad \left(v \cup w\right)^{*} = v^{*} \cup w^{*}$

2. (30 points) Assume we are given NFA-λ $M_1 = (Q_1, \Sigma, \delta_1, q_{01}, F_1)$ which accepts $L_1 = L(M_1)$ and DFA $M_2 = (Q_2, \Sigma, \delta_2, q_{02}, F_2)$ which accepts $L_2 = L(M_2)$.

a) Must $(L_1 \cup L_2)L_2^*$ be regular? Justify your response.

b) Describe precisely an NFA-λ to accept L_1L_2.

3. (33 points) Consider the language $L = b(ab)^*$.

a) Describe a regular grammar *G* to generate *L*.

b) Show a derivation of *babab* using your grammar.

c) Show the derivation tree corresponding to your derivation in part *b*).

4 (25 points) Prove or give a counterexample to the following

CONJECTURE: For any regular grammar G and any $z \in L(G)$, any derivation of z is always a leftmost derivation and a rightmost derivation.

CS2022/MA2201

Solutions to Midterm Exam

1. *a*) true

b) false If v=a and w=b, then $ba \in w(vw \cup w)^* v$ but $ba \notin vv^* w(vv^* w)^*$

c) false If v=a and w=b, then $ba \in (v \cup w)^*$ but $ba \notin v^* \cup w^*$

2.(*a*) Because L_1 is accepted by a NFA-λ and L_2 is accepted by a DFA, then they must be regular. We also showed in class and in the text that the union and *-closure of regular languages must be regular, so $L_1 \cup L_2$ and L_2^* must be regular. Likewise, the concatenation of $L_1 \cup L_2$ and L_2^* must be regular.

b) $L_1 L_2$ is accepted by $M_3 = (Q_1 \cup Q_2, \Sigma, \delta_3, q_{01}, F_2)$ where

$(\forall q \in Q_1)(\forall a \in A)\delta_3(q,a) = \delta_1(q,a)$

$(\forall q \in F_1)\delta_3(q,\lambda) = \delta_1(q,\lambda) \cup \{q_{02}\}$

$(\forall q \in Q_1 - F_1)\delta_3(q,\lambda) = \delta_1(q,\lambda)$

$(\forall q \in Q_2)(\forall a \in A)\delta_3(q,a) = \delta_2(q,a)$

$(\forall q \in F_2)\delta_3(q,\lambda) = \delta_2(q,\lambda)$

3. *a*)

$S \to bA$

$A \to \lambda | aB$

$B \to bA$

b) $S \Rightarrow bA \Rightarrow baB \Rightarrow babA \Rightarrow babaB \Rightarrow bababA \Rightarrow babab$

c)

```
        S
       / \
      b   A
         / \
        a   B
           / \
          b   A
             / \
            a   B
               / \
              b   A
                  |
                  λ
```

4. The CONJECTURE is true. Because G is regular, every sentential form in a derivation has at most one variable, which must be the leftmost and the rightmost variable.