

CS3133
Midterm Exam

Name _____

Date: September 25, 2000

All documentation permitted

1. (12 points) Tell whether each of the following is true or false for all regular expressions v and w . For each identity you believe to be false, give examples for v and w for which it is false.

a) $(v^* w^*)^* = (v \cup w)^*$

b) $w(vw \cup w)^* v = vv^* w(vv^* w)^*$

c) $(v \cup w)^* = v^* \cup w^*$

2. (30 points) Assume we are given NFA- λ $M_1 = (Q_1, \Sigma, \delta_1, q_{01}, F_1)$ which accepts $L_1 = L(M_1)$ and DFA $M_2 = (Q_2, \Sigma, \delta_2, q_{02}, F_2)$ which accepts $L_2 = L(M_2)$.

a) Must $(L_1 \cup L_2)L_2^*$ be regular? Justify your response.

b) Describe precisely an NFA- λ to accept L_1L_2 .

3. (33 points) Consider the language $L = b(ab)^*$.

a) Describe a regular grammar G to generate L .

b) Show a derivation of $babab$ using your grammar.

c) Show the derivation tree corresponding to your derivation in part b).

4 (25 points) Prove or give a counterexample to the following

CONJECTURE: For any regular grammar G and any $z \in L(G)$, any derivation of z is always a leftmost derivation and a rightmost derivation.

CS2022/MA2201
Solutions to Midterm Exam

1. a) true

b) false If $v=a$ and $w=b$, then $ba \in w(vw \cup w)^* v$ but $ba \notin vv^* w(vv^* w)^*$

c) false If $v=a$ and $w=b$, then $ba \in (v \cup w)^*$ but $ba \notin v^* \cup w^*$

2.(a) Because L_1 is accepted by a NFA- λ and L_2 is accepted by a DFA, then they must be regular. We also showed in class and in the text that the union and *-closure of regular languages must be regular, so $L_1 \cup L_2$ and L_2^* must be regular. Likewise, the concatenation of $L_1 \cup L_2$ and L_2^* must be regular.

b) $L_1 L_2$ is accepted by $M_3 = (Q_1 \cup Q_2, \Sigma, \delta_3, q_{01}, F_2)$ where

$$(\forall q \in Q_1)(\forall a \in A) \delta_3(q, a) = \delta_1(q, a)$$

$$(\forall q \in F_1) \delta_3(q, \lambda) = \delta_1(q, \lambda) \cup \{q_{02}\}$$

$$(\forall q \in Q_1 - F_1) \delta_3(q, \lambda) = \delta_1(q, \lambda)$$

$$(\forall q \in Q_2)(\forall a \in A) \delta_3(q, a) = \delta_2(q, a)$$

$$(\forall q \in F_2) \delta_3(q, \lambda) = \delta_2(q, \lambda)$$

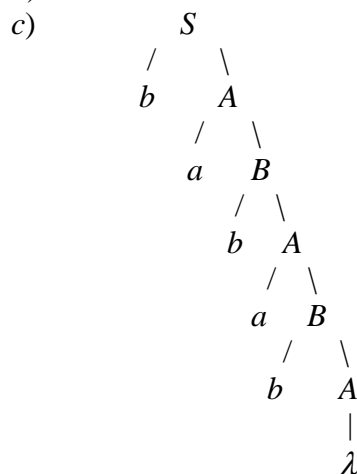
3. a)

$$S \rightarrow bA$$

$$A \rightarrow \lambda | aB$$

$$B \rightarrow bA$$

b) $S \Rightarrow bA \Rightarrow baB \Rightarrow babA \Rightarrow babaB \Rightarrow bababA \Rightarrow babab$



4. The CONJECTURE is true. Because G is regular, every sentential form in a derivation has at most one variable, which must be the leftmost and the rightmost variable.