1. (12 points) Tell whether each of the following is true or false for all regular expressions \( v \) and \( w \). For each identity you believe to be false, give examples for \( v \) and \( w \) for which it is false.

   a) \( (v^*w^*)^* = (v \cup w)^* \)

   b) \( w(vw \cup w)^* v = vv^*w(vv^*w)^* \)

   c) \( (v \cup w)^* = v^* \cup w^* \)
2. (30 points) Assume we are given NFA- $\lambda$ $M_1 = (Q_1, \Sigma, \delta_1, q_{01}, F_1)$ which accepts $L_1 = L(M_1)$ and DFA $M_2 = (Q_2, \Sigma, \delta_2, q_{02}, F_2)$ which accepts $L_2 = L(M_2)$.

   a) Must $(L_1 \cup L_2) L_2^*$ be regular? Justify your response.

   b) Describe precisely an NFA- $\lambda$ to accept $L_1 L_2$. 


3. (33 points) Consider the language \( L = b(ab)^* \).

a) Describe a regular grammar \( G \) to generate \( L \).

b) Show a derivation of \( babab \) using your grammar.

c) Show the derivation tree corresponding to your derivation in part b).
4 (25 points) Prove or give a counterexample to the following

**Conjecture**: For any regular grammar $G$ and any $z \in L(G)$, any derivation of $z$ is always a leftmost derivation and a rightmost derivation.
1. a) true
b) false If \( v = a \) and \( w = b \), then \( ba \in w(vw \cup w)^* \) but \( ba \notin v^* w(v^* w)^* \)
c) false If \( v = a \) and \( w = b \), then \( ba \in (v \cup w)^* \) but \( ba \notin v^* \cup w^* \)

2. (a) Because \( L_1 \) is accepted by a NFA- \( \lambda \) and \( L_2 \) is accepted by a DFA, then they must be regular. We also showed in class and in the text that the union and \( * \)-closure of regular languages must be regular, so \( L_1 \cup L_2 \) and \( L_2^* \) must be regular. Likewise, the concatenation of \( L_1 \cup L_2 \) and \( L_2^* \) must be regular.

b) \( L_1 L_2 \) is accepted by \( M_3 = (Q_1 \cup Q_2, \Sigma, \delta_3, q_{01}, F_2) \) where
\[
\begin{align*}
(\forall q \in Q_1)(\forall a \in \Sigma)\delta_3(q, a) & = \delta_1(q, a) \\
(\forall q \in F_1)\delta_3(q, \lambda) & = \delta_1(q, \lambda) \cup \{q_{02}\} \\
(\forall q \in Q_1 - F_1)\delta_3(q, \lambda) & = \delta_1(q, \lambda) \\
(\forall q \in Q_2)(\forall a \in \Sigma)\delta_3(q, a) & = \delta_2(q, a) \\
(\forall q \in F_2)\delta_3(q, \lambda) & = \delta_2(q, \lambda)
\end{align*}
\]

3. a) \( S \rightarrow bA \)
    \( A \rightarrow \lambda \mid aB \)
    \( B \rightarrow bA \)

b) \( S \Rightarrow bA \Rightarrow baB \Rightarrow babA \Rightarrow babab \Rightarrow babab \Rightarrow babab \)

c) \[
\begin{array}{c}
S \\
/ \ \\
b \ \\
/ \ \\
a \ B \\
/ \ \\
b \ A \\
/ \ \\
a \ B \\
/ \ \\
b \ A \\
/ \ \\
\lambda
\end{array}
\]

4. The CONJECTURE is true. Because \( G \) is regular, every sentential form in a derivation has at most one variable, which must be the leftmost and the rightmost variable.