CS3133

FINAL EXAM

Name______________

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All documentation permitted

1. (10 points) Prove or give a counterexample to the following conjecture.

CONJECTURE: The complement of any Context-Free Language is not a Context-Free Language.

2. (40 points) For each of the following languages, tell whether or not it is Context-Free. Justify your answer.

a $L = \{a^i \mid i \geq 0\} \cup \{a^i b^i \mid i \geq 0\} \cup \{a^i b^k c^i \mid i \geq 0, k \geq 2\}$

b L is the set of all strings over $\Sigma = \{a, b, c\}$ such that every prefix of every string in L contains at least as many c's as the number of a's plus the number of b's. Thus, $ccabccb \in L$ and $\varepsilon \in L$ but $ccabccbbacc \notin L$. Note that $ccabccbbacc \notin L$ because it contains the prefix $ccabccbba$ which has five a's and b's but only four c's.

3. (25 points) Is the following CONJECTURE true? Justify your answer.

CONJECTURE: There is a constant *c* such that for every Context-Free Language *L*, there is a Context-Free Grammar *G* such that for every string $z \in L$ there is a derivation of *z* in *G* of length at most $c * |z|$.

4. (25 points) Are recursive languages closed under concatenation? That is, if L_0 and L_1 are recursive, must L_0L_1 be recursive? Justify your answer.

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Solutions to FINAL EXAM

1. The CONJECTURE is false. The languages $\varnothing$ and $(0+1)^*$ over $\Sigma = \{0,1\}$ are each Context-Free, because they admit the grammars $G_\varnothing$ with no productions and $G_{(0+1)^*}$ with productions $S \to 0S|1S|\varepsilon$, and each is the complement of the other.

2. ***a*** $\{a^i | i \geq 0\} \cup \{a^i b^i | i \geq 0\} \cup \{a^i b^j c^i | i \geq 0, j \geq 2\}$ is a CFL. It is generated by

$$S \to S_1|S_2|S_3$$
$$S_1 \to aS_1|\varepsilon$$
$$S_2 \to aS_2b|\varepsilon$$
$$S_3 \to aS_4c|S_4$$
$$S_4 \to bS_4|bb$$

b We accept the language by final state with a one state PDA. State q is the start state and the final state.

$$\delta(q,c,\perp) = \{(q,c\perp)\}$$
$$\delta(q,c,c) = \{(q,cc)\}$$
$$\delta(q,a,c) = \{(q,\varepsilon)\}$$
$$\delta(q,b,c) = \{(q,\varepsilon)\}$$

3. The CONJECTURE is true. We know that every CFL L admits a CFG G in CNF to generate $L - \{\varepsilon\}$. We know that if G is in CNF, then every production either makes the sentential form one longer (starting with S this can happen $|z|-1$ times) or adds a terminal (this can happen $|z|$ times). So we choose c=2.

4. Yes, recursive languages are closed under concatenation. Assume that L_0 and L_1 are recursive. To test if $a_1 ... a_n \in L_0 L_1$ we execute

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if a_1...a_n ∈ L_0 ∧ ε ∈ L_1 or ε ∈ L_0 ∧ a_1...a_n ∈ L_1 then return accept
for i ← 1 to n-1 do
    if a_1...a_i ∈ L_0 and a_{i+1}...a_n ∈ L_1 then return accept
return reject
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