CS3133

FINAL EXAM

Name______________

Date: October 11, 2007

All documentation permitted

1. (25 points) Is the following language context free? Justify your answer.

$$\left\{a^m b^n c^p \mid \left(m = n \text{ or } n = p\right) \text{ and } \left(m, n, p \geq 0\right)\right\} .$$

2. (25 points) Are recursive languages closed under intersection? That is, for any recursive languages L_1 and L_2, must $L_1 \cap L_2$ be recursive? Explain why or why not.

3. (25 points) Construct a CFG in Chomsky Normal Form for the language $\left\{a^m b^{2m} c^n \mid m, n \geq 1\right\}$.

4. (25 points) Is it decidable whether a string does **not** belong to a context free language? That is, given CFG G and string $w \in \Sigma^*$, is it decidable if $w \notin L(G)$?

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Solutions to Final Exam

1. $\{a^m b^n c^p \mid m = n \text{ or } n = p\} = \{a^m b^n c^p \mid m = n\} \cup \{a^m b^n c^p \mid n = p\} = L_1 \cup L_2$

A grammar to generate L_1 is

$$S_1 \to A_{m=n} A_{stringcs}$$
$$A_{m=n} \to \varepsilon \mid aA_{m=n}b$$
$$A_{stringcs} \to \varepsilon \mid cA_{stringcs}$$

and a grammar to generate L_2 is

$$S_2 \to A_{stringas} A_{n=p}$$
$$A_{stringas} \to \varepsilon \mid aA_{stringas}$$
$$A_{n=p} \to \varepsilon \mid bA_{n=p}c$$

And finally $S \to S_1 \mid S_2$.

2. Because L_1 and L_2 are recursive, there are total Turing Machines M_1 and M_2 such that $L(M_1) = L_1$ and $L(M_2) = L_2$. So there is a total Turing Machine

if $w \in L(M_1)$ **and** $w \in L(M_2)$ **then return** accept
else return reject

to accept $L_1 \cap L_2$.

3. A CFG to generate $\{a^m b^{2m} c^n \mid m, n \geq 1\}$ is

$$S \to A_{a^m b^{2m}} A_{c^n}$$
$$A_{a^m b^{2m}} \to aA_{a^m b^{2m}} bb \mid abb$$
$$A_{c^n} \to cA_{c^n} \mid c$$

Putting this into Chomsky Normal Form,

$$S \to A_{a^m b^{2m}} A_{c^n}$$
$$A_{a^m b^{2m}} \to A_a X_0 \mid A_a X_1$$
$$X_0 \to A_{a^m b^{2m}} X_1$$
$$X_1 \to B_b B_b$$
$$A_a \to a$$
$$B_b \to b$$
$$A_{c^n} \to C_c A_{c^n} \mid c$$
$$C_c \to c$$

4. The question is decidable. Given CFG G, we can convert it to an equivalent CFG in Chomsky Normal Form. Testing all derivations in the new grammar of length $2*|w|-1$, we return $w \notin L(G)$ if and only if there does not exist a derivation of w.