CS3133

FINAL EXAM

Name______________

Date: October 13, 2005

All documentation permitted

1. (25 points) Either construct a PDA to accept the language generated by

$$S \to ASA \mid Aa$$

$$A \to AAS \mid a$$

or prove that there doesn't exist such a PDA.

2. (25 points) Show that for any recursive language $L \subseteq \Sigma^*$, it is decidable whether $z \in \Sigma^*$ is a shortest string in *L*. That is, given (L, z), *L* recursive, there is a procedure to decide if $z \in L$ and if there exists a $y \in L$ with $|y| < |z|$.

3. (25 points) Is the following question decidable:

INPUT: NFA $N = (Q, \Sigma, \Delta, S, F)$.

QUESTION: Is $L(N) = \Sigma^*$?

That is, can we decide if there is a string **not** accepted by an arbitrary NFA?

4. (25 points) Is $L = \left\{ 0^n 1 0^{2n} 1 0^{3n} \mid n \geq 0 \right\}$ context-free? Justify your response.

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Solutions to <u>Final Exam</u>

1. We first express the grammar in Chomsky Normal Form

$$S \to AY_1 \mid AA_a$$
$$Y_1 \to SA$$
$$A_a \to a$$
$$A \to AY_2 \mid a$$
$$Y_2 \to AS$$

The language is accepted by empty stack by the PDA

$M = \left(\{q\}, \{a\}, \{\perp, S, A, A_a, Y_1, Y_2\}, \delta, S, \perp, \varnothing\right)$, where

$$\delta(q, \varepsilon, S) = \{(q, AY_1), (q, AA_a)\}$$
$$\delta(q, \varepsilon, Y_1) = \{(q, SA)\}$$
$$\delta(q, a, A_a) = \{(q, \varepsilon)\}$$
$$\delta(q, \varepsilon, A) = \{(q, AY_2)\}$$
$$\delta(q, a, A) = \{(q, \varepsilon)\}$$
$$\delta(q, \varepsilon, Y_2) = \{(q, AS)\}$$

2. Because L is recursive, there exists a total Turing Machine M such that $L(M) = L$.
Also, for any $z \in \Sigma^*$, there are only a finite number of $y \in \Sigma^*$ with $|y| < |z|$.

ShortestinL? ← true
for each $y \in \Sigma^*$, $|y| < |z|$ **do**
  if *M* accepts *y* **then** *ShortestinL?* ← false
return (*ShortestinL?* **and** *M* accepts *z*)

3. The question is decidable. We have seen an algorithm to convert N to a DFA $M = \left(Q^*, \Sigma, \delta, s, F^*\right)$, and $L(M)=L(N)=\Sigma^*$ if and only if every state in Q^* reachable from s belongs to F^*.

4. We show that L is not context-free by using the Pumping Lemma for context-free languages. We let k be the constant guaranteed by the Pumping Lemma, and let $z = 0^k10^{2k}10^{3k}$. By the Pumping Lemma, we must be able to write $z = uvwxy$ with $|vwx| \le k, |vx| \ge 1$. So vx must contain 0's from at least one of the strings 0^k, 0^{2k} and 0^{3k} and at most two of the strings 0^k, 0^{2k} and 0^{3k}. So pumping v and x in uv^2wx^2y will increase the numner of 0's in at least one but fewer than three of the strings 0^k, 0^{2k} and 0^{3k}. So

$uv^2wx^2y \notin L$, though the Pumping Lemma states that $uv^2wx^2y \in L$. By contradiction, $\{0^n10^{2n}10^{3n} \mid n \geq 0\}$ is not a CFL.