CS3133

Final Exam

Name______________

Date: October 10, 2002

All documentation permitted

1. (20 points) Consider the context free grammar G with productions

$$S \to aAA$$

$$A \to aS \mid bS \mid a$$

Either describe a PDA P to accept $L(G)$ or prove that none exists.

2. (30 points) Give a context free grammar in Chomsky Normal Form to generate

$$\left\{a^i b^j c^k \mid i \geq 1 \wedge k > j > 0\right\}.$$

3. (25 points) Prove or give a counterexample to the following

<u>CONJECTURE</u>: If $G_1 = (V_1, T, P_1, S_1)$ and $G_2 = (V_2, T, P_2, S_2)$ are context free grammars, then $L(G_1) \cap L(G_2)$ is never a context free language.

4 (25 points) Describe a Turing Machine to accept the language $0(0+1)^{*}1$.

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Solutions to Final Exam

1. There is a PDA to accept every CFL. $P=\left(\{q\},\{a,b\},\{S,A,a,b\},d,q,S\right)$ where $d\left(q,a,a\right)=\left(q,e\right)$, $d\left(q,b,b\right)=\left(q,e\right)$, $d\left(q,e,S\right)=\left(q,aAA\right)$, $d\left(q,e,A\right)=\left\{\left(q,aS\right),\left(q,bS\right),\left(q,a\right)\right\}$ accepts $L(G)$ by empty stack.

2. A CFG to generate $\left\{a^i b^j c^k \mid i\geq 1 \wedge k > j > 0\right\}$ is

$S \rightarrow AB$ $\quad$ A generates a^i, B generates $b^j c^k$

$A \rightarrow aA|a$ $\quad$ A generates a nonempty string of a's

$B \rightarrow CD$

$C \rightarrow bCc|bc$ $\quad$ C generates $b^j c^j$

$D \rightarrow cD|c$ $\quad$ D generates a nonempty string of c's

To put this in CNF, we derive

$$
\begin{aligned}
&S \rightarrow AB\\
&A \rightarrow X_a A|a\\
&X_a \rightarrow a\\
&B \rightarrow CD\\
&C \rightarrow X_b Y_1 | X_b X_c\\
&Y_1 \rightarrow C X_c\\
&X_b \rightarrow b\\
&D \rightarrow X_c D | X_c\\
&X_c \rightarrow c
\end{aligned}
$$

3. The conjecture is false. If $G_1=\left(\{S_1\},\{a\},\varnothing,S_1\right)$ and $G_2=\left(\{S_2\},\{a\},\{S\rightarrow e, S\rightarrow aS\},S_2\right)$, then $L\left(G_1\right)=\varnothing$ and $L\left(G_2\right)=T^*=a^*$, but $L\left(G_1\right)\cap L\left(G_2\right)=\varnothing=L\left(G_1\right)$, which is certainly context free. An even easier counterexample for CFGs G_1 and G_2 is to choose $L=L\left(G_1\right)\cap L\left(G_1\right)=L\left(G_1\right)$.

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4. $M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \{0,1,B\}, d, q_0, B, \{q_3\})$

where $d(q_0, 0) = (q_1, 0, R)$ make sure first symbol is 0

$d(q_1, 0) = (q_1, 0, R)$ go right until a B

$d(q_1, 1) = (q_1, 1, R)$ go right until a B

$d(q_1, B) = (q_2, 1, L)$ found a B, go check symbol to the left

$d(q_2, 1) = (q_3, 1, R)$ if it's a 1, then accept