CS3133

Final Exam

Name_____________

Date: October 19, 2000

All documentation permitted

1. (25 points) Either construct a regular grammar to show that $\left\{a^i b^j \mid i, j \in \mathbb{N} \wedge i \leq j\right\}$ is regular or use the Pumping Lemma for regular languages to show that it is not regular.

2. (25 points) Either construct a context-free grammar to show that $\{a^i b^i a^j b^j b^k \mid i, j, k \in \mathbb{N}\}$ is context-free or use the Pumping Lemma for context-free languages to show that it is not context-free.

3. (25 points) Let G be a grammar in Greibach Normal Form and let $z \in L(G)$ with $length(z) = n$. Provide bounds for the number of steps in a derivation $S \overset{*}{\Rightarrow} z$. That is, you should provide the tightest lower and upper bounds possible on the length of a derivation of z. That is, what is the largest value of p such that every derivation $S \overset{*}{\Rightarrow} z$ must have at least p steps, and what is the smallest value of q such that every derivation $S \overset{*}{\Rightarrow} z$ must have at most q steps

4. (25 points) Prove that $a^*b(a \cup b)^* \in NP$.

CS3133

Solutions to Final Exam

1. $\{a^i b^j \mid i, j \in \mathbb{N} \wedge i \le j\}$ is not regular. Assume it is regular, and let n be the constant provided by the Pumping Lemma and let $z=a^n b^n$. If L were regular, then z could be written $uvw = a^n b^n$ with $length(uv) \le n$ and $length(v) \ge 1$. It follows that v must be a nonempty string of as. The Pumping Lemma asserts that $uv^2w \in L$, even though it has more as than bs. This contradiction implies that $\{a^i b^j \mid i, j \in \mathbb{N} \wedge i \le j\}$ is not regular.

2. $\{a^i b^i a^j b^j b^k \mid i, j, k \in \mathbb{N}\}$ is context-free because it is generated by the grammar

$$S \to AAB$$
$$A \to aAb \mid \lambda$$
$$B \to bB \mid \lambda$$

3. The upper and lower bounds are both $length(z) = n$, unless $\lambda \in L(G)$, in which case the upper bound is $n+1$. Since every step in a derivation (aside from $S \Rightarrow \lambda$) adds exactly one terminal symbol to the sentential form, all such derivations last exactly $length(z) = n$ steps.

4. $a^* b (a \cup b)^*$ is accepted by final state in $O(n)$ time (where n is the length of the input string) by the nondeterministic Turing Machine

$$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b, B\}, \delta, q_0, \{q_2\})$$

with transition function

δ	B	a	b
q_0	$\{[q_1, B, R]\}$	$\varnothing$	$\varnothing$
q_1	$\varnothing$	$\{[q_1, a, R]\}$	$\{[q_2, b, R]\}$
q_2	$\varnothing$	$\varnothing$	$\varnothing$