CS3133
HW#7 SOLUTIONS

1. Removing unit productions yields

$$S \to \text{if } C \text{ then } A \mid \text{while } C \text{ do } A \mid \text{if } C \text{ then } A \text{ else } S$$
$$B \to \text{if } C \text{ then } A \text{ else } S$$
$$C \to c_1 \mid c_2 \mid c_3$$
$$A \to a_1 \mid a_2 \mid a_3$$

Since B is now unreachable, removing useless productions now yields

$$S \to \text{if } C \text{ then } A \mid \text{while } C \text{ do } A \mid \text{if } C \text{ then } A \text{ else } S$$
$$C \to c_1 \mid c_2 \mid c_3$$
$$A \to a_1 \mid a_2 \mid a_3$$

Finally, we can convert to Chomsky Normal Form

$$S \to X_1Y_1 \mid X_3Y_3 \mid X_1Y_5$$
$$X_1 \to \text{if}$$
$$Y_1 \to CY_2$$
$$Y_2 \to X_2A$$
$$X_2 \to \text{then}$$
$$X_3 \to \text{while}$$
$$Y_3 \to CY_4$$
$$Y_4 \to X_4A$$
$$X_4 \to \text{do}$$
$$Y_5 \to CY_6$$
$$Y_6 \to X_2Y_7$$
$$Y_7 \to AY_8$$
$$Y_8 \to X_5S$$
$$X_5 \to \text{else}$$
$$C \to c_1 \mid c_2 \mid c_3$$
$$A \to a_1 \mid a_2 \mid a_3$$

2. Let n be given by the PUMPING LEMMA, and consider $z = 0^n1^n1^n0^n0^n1^n = 0^n1^{2n}0^{2n}1^n$. When z is rewritten as $uvwxy$ with $|vwx| \leq n$ and $|vx| \geq 1$, then vx contains at least a 0 or at least a 1, and it can not contain 0's from both clumps of 0's nor can it contain 1's from both clumps of 1's. Then uv^2wx^2y no longer has twice as many 0's in the second clump as the first or no longer has twice as many 1's in the first clump as the second.

3. Let n=42. For every $z \in L, |z| \geq 42$, we must be able to express z as $uvwxy$ which satisfies certain conditions. Since there is no such z, these conditions are vacuously satisfied, and we can **not** create a contradiction.

4. **a)** $L(G) = \emptyset$ if and only if S is generating. In fact, since useless symbols have been removed from G, we can conclude that $L(G) = \emptyset$ if and only if $P = \emptyset$.

b) Yes, $L(G)$ must be infinite. Since G has no useless symbols, A is reachable, $S \overset{*}{\Rightarrow} a A b$ $a, b \in (V \cup T)^*$, $w \in T^*$, and generating, $A \overset{*}{\Rightarrow} w$, $w \in T^*$. Also, there are derivations $a \overset{*}{\Rightarrow} u$, $u \in T^*$ and $b \overset{*}{\Rightarrow} y$, $y \in T^*$, so $S \overset{*}{\Rightarrow} a A b \overset{*}{\Rightarrow} uwy$. But also $A \overset{+}{\Rightarrow} g A V$, $g, V \in (V \cup T)^*$, and $g \overset{*}{\Rightarrow} v$, $v \in T^*$ and $V \overset{*}{\Rightarrow} x$, $x \in T^*$ where $|vx| \geq 1$. It follows that $S \overset{*}{\Rightarrow} uwy$, $S \overset{*}{\Rightarrow} uvwxy$, $S \overset{*}{\Rightarrow} uvvwxxy$,..., and $\{uv^k wx^k y \mid k \geq 0\} \subseteq L(G)$.