CS3133
HW #7 Solutions

1. *a*) L_1 is generated by

$$S \to AC$$
$$A \to aAbb|\lambda$$
$$C \to cC|\lambda$$

b) L_2 is generated by

$$S \to AB$$
$$A \to aA|\lambda$$
$$B \to bBcc|\lambda$$

c) $L_1 \cap L_2 = \{a^i b^{2i} c^{4i} | i \in \mathbb{N}\}$ is not context-free. Assume it is context-free, and let $z = a^k b^{2k} c^{4k}$ where k is the number guaranteed by the pumping lemma for context-free languages. Expressing $z = uvwxy$ as in the pumping lemma, since $length(vwx) \leq k$ it follows that vx can not span as, bs **and** cs, and since $length(v) + length(x) \geq 1$ it follows that vx must span as, bs **or** cs. "Pumping up" vx (since $\{uv^i wx^i y | i \in \mathbb{N}\} \subseteq L_1 \cap L_2$) would thus cause a contradiction, since the cardinalities of **some** (but not **all**) of $\{a,b,c\}$ would be changed.

2. If L is context-free, there is a context-free grammar $G = (V, \Sigma, P, S)$ which generates L. The grammar $G' = (V, \Sigma - \{a\}, P', S)$, where P' is obtained from P by erasing all as from the righthand side of all rules of P, generates $er_a(L)$, that is, $er_a(L) = L(G')$.

3. *a*) $(a \cup c)^* cc(a \cup c)^* b(a \cup b \cup c)^*$

b) $M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b, c\}, \{a, b, c, B\}, \delta, q_0, \{q_4\})$ where

δ	B	a	b	c
q_0	q_1, B, R			
q_1		q_1, a, R		q_2, c, R
q_2		q_1, a, R		q_3, c, R
q_3		q_3, a, R	q_4, b, R	q_3, c, R