CS3133

HW#7

DUE: Tuesday, October 9

1. (8 points) Describe a Turing Machine to accept L, the set of all binary strings with a length divisible by 3. For example $110010 \in L$ and $\varepsilon \in L$ though $0101 \notin L$.

2. (8 points) Are the following questions decidable?

a INPUT: Alphabet Σ, string $w \in \Sigma^*$, CFGs G_1, G_2 over Σ.
QUESTION 1: Does $w \in L(G_1) \cap L(G_2)$?

b INPUT: Alphabet Σ, CFG G over Σ.
QUESTION 1: Does $\varepsilon \in L(G)$?

3. (10 points) Prove that language L is recursive if and only if we can enumerate it in order of nondecreasing length. That is, L is recursive if and only if we can list $z_0, z_1, ...$ such that $(x \in L) \leftrightarrow (\exists i)\, x = z_i$ and $(i > j) \rightarrow (|z_i| \geq |z_j|)$.

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Solutions for HW#7

1. We let *b* denote the blank tape symbol.

$$\delta(s,\vdash) = (q_0,\vdash,R)$$
$$\delta(q_0,0) = \delta(q_0,1) = (q_1,0,R)$$
$$\delta(q_1,0) = \delta(q_1,1) = (q_2,0,R)$$
$$\delta(q_2,0) = \delta(q_0,1) = (q_0,0,R)$$
$$\delta(q_0,b) = \delta(t,b) = (t,b,R)$$
$$\delta(q_1,b) = \delta(q_2,b) = \delta(r,b) = (r,b,R)$$

2. The questions are each decidable.

a We can convert G_1 and G_2 to equivalent grammars in Chomsky Normal form. We can then try all derivations of length $2|w|-1$ is each grammar to test if *w* belongs to the language that each grammar generates.

b $\varepsilon \in L(G)$ if and only is $S \in$ NULLABLE, the set of nonterminals which can be erased.

3. If we can list the members of *L* in order of nondecreasing lengths, then to test whether a string *x* belongs to *L* we just examine the list until either we find a member with length greater than $|x|$ (in which case we reject *x*) or we encounter *x* (in which case we accept it).

If *L* is recursive, then there is a total Turing machine *M* to accept it. The following machine lists *L*.

for $n \leftarrow 0$
  for each $z \in \Sigma^*, |z| = n$
    if $z \in L(M)$ **then** "output *z*"
  $n \leftarrow n+1$