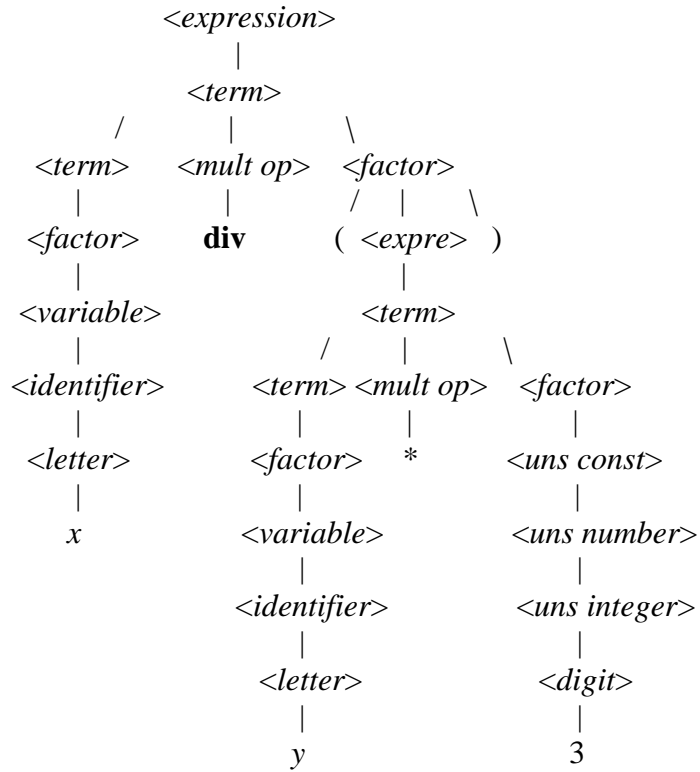


CS3133
HW #6 SOLUTIONS

1.



2. An equivalent grammar without useless symbols is

$$S \rightarrow CA$$

$$A \rightarrow a$$

$$C \rightarrow b$$

which is equivalent to the trivial grammar $S \rightarrow ab$.

3. An equivalent essentially noncontracting grammar without chain rules is

$$S \rightarrow bA | b$$

$$A \rightarrow AbA | bA | Ab | b | aS | a | aB$$

$$B \rightarrow aB | a$$

An equivalent grammar in Chomsky Normal Form is

$$\begin{aligned}
S &\rightarrow Y_1 A | b \\
Y_1 &\rightarrow b \\
A &\rightarrow AZ_2 | Y_3 A | AY_4 | b | Y_5 S | a | Y_6 B \\
Z_2 &\rightarrow Y_2 A \\
Y_2 &\rightarrow b \\
Y_3 &\rightarrow b \\
Y_4 &\rightarrow b \\
Y_5 &\rightarrow a \\
Y_6 &\rightarrow a \\
B &\rightarrow Y_7 B | a \\
Y_7 &\rightarrow a
\end{aligned}$$

4. The grammar is already in Chomsky Normal Form. After the first phase of the algorithm, the grammar is

$$\begin{aligned}
S &\rightarrow AB \\
A &\rightarrow BS | b \\
B &\rightarrow bBAZ_3 | aZ_3 | bBA | a \\
Z_3 &\rightarrow SBA | SBAZ_3
\end{aligned}$$

Replacing the A and the B on the righthand sides of the first two rules yields

$$\begin{aligned}
S &\rightarrow bBAZ_3 SB | aZ_3 SB | bBASB | aSB | bB \\
A &\rightarrow bBAZ_3 S | aZ_3 S | bBAS | aS | b \\
B &\rightarrow bBAZ_3 | aZ_3 | bBA | a \\
Z_3 &\rightarrow SBA | SBAZ_3
\end{aligned}$$

“Fixing” the two Z_3 productions (replacing them with ten! new rules) yields:

$$\begin{aligned}
S &\rightarrow bBAZ_3 SB | aZ_3 SB | bBASB | aSB | bB \\
A &\rightarrow bBAZ_3 S | aZ_3 S | bBAS | aS | b \\
B &\rightarrow bBAZ_3 | aZ_3 | bBA | a \\
Z_3 &\rightarrow bBAZ_3 SBBA | aZ_3 SBBA | bBASBBA | aSBBA | bBBA \\
Z_3 &\rightarrow bBAZ_3 SBBAZ_3 | aZ_3 SBBAZ_3 | bBASBBAZ_3 | aSBBAZ_3 | bBBAZ_3
\end{aligned}$$