CS3133

HW#6

DUE: Friday, September 30

1. (16 points) For each of the following languages, tell whether or not it is context-free and justify your answers.

a $\left\{a^i b^j c^k \mid 0 \le i \le j \le k\right\}$.

b $\left\{a^i b^j c^k \mid i \ne j \vee j = k\right\}$.

2. (10 points) Prove that if L_0 is a regular language and L_1 is a CFL, then $L_0 \cap L_1$ must be a CFL.

CS3133

Solutions to HW#6

1. ***a*** $L=\left\{a^i b^j c^k \mid 0 \le i \le j \le k\right\}$ is not a CFL. We prove this by assuming that it is context-free and deriving a contradiction from the Pumping Lemma for CFLs. Let *n* be as provided by the PL for CFLs, and choose $z=a^n b^n c^n$. There must exist a decomposition $z=uvwxy$ guaranteed by the PL. There are two cases to consider:

- If either *v* or *x* contains 2 types of symbols, then uv^2wx^2y contains *a*'s, *b*'s and *c*'s in the wrong order. So $uv^2wx^2y \notin L$, although the PL says it must. So this is a contradiction.
- If each of *v* and *x* contains at most 1 type of symbol, there are three subcases:
  - If *vx* doesn't have any *a*'s, then *uwy* has more *a*'s than *b*'s or more *a*'s than *c*'s. So $uv^0wx^0y = uwy \notin L$ $uv^2wx^2y \notin L$, although the PL says it must. So this is a contradiction.
  - If *vx* doesn't have any *b*'s, then if *vx* has *a*'s, then uv^2wx^2y has more *a*'s than *b*'s. If *vx* has *c*'s, then $uv^0wx^0y = uwy$ has more *b*'s than *c*'s. In either case, we get a contradiction from the PL
  - If *vx* doesn't have any *c*'s, then uv^2wx^2y either has more *a*'s than *c*'s or more *b*'s than *c*'s. In either case, we get a contradiction from the PL.

b $\left\{a^i b^j c^k \mid i \ne j \vee j = k\right\}$ is a CFL because it is the union of 2 CFLs, $\left\{a^i b^j c^k \mid i \ne j\right\}$ and $\left\{a^i b^j c^k \mid j = k\right\}$, and we know that CFLs are closed under union. It is easy to see that $\left\{a^i b^j c^k \mid i \ne j\right\}$ is generated by the CFG:

$$S \to S_{i>j} \mid S_{i<j}$$
$$S_{i>j} \to AS_{i=j}C$$
$$A \to aA \mid a$$
$$S_{i=j} \to aS_{i=j}b \mid \varepsilon$$
$$C \to cC \mid \varepsilon$$
$$S_{i<j} \to S_{i=j}BC$$
$$B \to bB \mid b$$

and that $\left\{a^i b^j c^k \mid j = k\right\}$ is generated by the CFG:

$$S \to AS'$$
$$A \to aA \mid \varepsilon$$
$$S' \to bS'c \mid \varepsilon$$

2. If L_0 is a regular language and L_1 is a CFL, then there are DFA $M_0 = (Q_0, \Sigma, \delta, s_0, F_0)$ and PDA $M_1 = (Q_1, \Sigma, \Gamma, \Delta, s_1, \perp, F_1)$ such that $L_0 = L(M_0)$ and $L_1 = L(M_1)$. We accept $L_0 \cap L_1$ by constructing a PDA $M^* = (Q_0 \times Q_1, \Sigma, \Gamma, \Delta^*, (s_0, s_1), \perp, F_0 \times F_1)$ which "simulates" M_0 and M_1 in parallel. That is, for each $q_k \in Q_0, q_l \in Q_1, a \in \Sigma, A \in \Gamma$,

$$\Delta^*((q_k, q_l), a, A) = \{((q_i, q_j), B_1 B_2 ... B_k) \mid \delta(q_k a) = q_i \wedge ((q_l, a, A), (q_j, B_1 B_2 ... B_k) \in \Delta)\}$$.