CS3133

HW#6

DUE: Thursday, October 4

1. (8 points) Let L be the set of binary strings that contain the same number of 0's as 1's. For example, $0110010011 \in L$ and $\varepsilon \in L$ but $01100100111 \notin L$. Is L context free? Defend your answer.

2. (8 points) Prove or give a counterexample to the following.

   CONJECTURE: For any context free languages L_0 and L_1, the language $L_0 \cup L_1$ must be context free.

3. (8 points) Give unambiguous grammars for the following two languages.

a $\left\{0^m 1^n 0^{m+n} \mid m \geq 0, n \geq 1\right\}$.

b $\left\{0^m 1^n 0^{m-n} \mid m \geq n \geq 0\right\}$.

4. (3 points) Show that the grammar $S \to 0S1 \mid SS \mid \varepsilon$ is ambiguous.

Extra Credit: Caution – Do **not** do this problem in lieu of doing regular problems. Describe the language generated by the grammar

$$S \to 0S1S \mid \varepsilon \quad .$$

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Solutions for HW#6

1. L is context free, and a PDA to accept it by empty stack is $M = (\{q_0, q, q_1\}, \{0,1\}, \{\perp, 0, 1\}, \delta, q, \perp, \emptyset)$. When M is in state q_0 (respectively q_1), more 0's (resp. 1's) have been read than 1's (resp. 0's). In either case, the stack contains all the excess 0's or 1's. When in state q, the same number of 0's as 1's have been read.

$$\delta(q_0,0,0) = \{(q_0,00)\},\ \delta(q_0,1,0) = \{(q_0,\varepsilon)\},$$
$$\delta(q_1,1,1) = \{(q_1,11)\},\ \delta(q_1,0,1) = \{(q_1,\varepsilon)\},$$
$$\delta(q_0,\varepsilon,\perp) = \delta(q_1,\varepsilon,\perp) = \{(q,\perp)\},$$
$$\delta(q,0,\perp) = \{(q_0,0\perp)\},\ \delta(q,1,\perp) = \{(q_1,1\perp)\},$$
$$\delta(q,\varepsilon,\perp) = \{(q,\varepsilon)\}.$$

2. The CONJECTURE is true. If L_0 and L_1 are context free, then there are context free grammars $G_0 = (N_0, \Sigma, P_0, S_0)$ and $G_1 = (N_1, \Sigma, P_1, S_1)$ such that $L(G_0) = L_0$ and $L(G_1) = L_1$. Assuming $N_0 \cap N_1 = \emptyset$ and $S \notin N_0 \cup N_1$, then

$$G = (N_0 \cup N_1 \cup \{S\}, \Sigma, P_0 \cup P_1 \cup \{S \to S_0 | S_1\}, S)$$

generates $L_0 \cup L_1$.

3. ***a*** We rewrite $\{0^m1^n0^{m+n} \mid m \ge 0, n \ge 1\}$ as $\{0^m1^n0^n0^m \mid m \ge 0, n \ge 1\}$. A CFG to generate it is

$$S \to 0S0 \mid A_{mid}$$
$$A_{mid} \to 1A_{mid}0 \mid 10$$

b We rewrite $\{0^m1^n0^{m-n} \mid m \ge n \ge 0\}$ as $\{0^{m-n}0^n1^n0^{m-n} \mid m \ge n \ge 0\}$. A CFG to generate it is

$$S \to 0S0 \mid A_{mid}$$
$$A_{mid} \to 0A_{mid}1 \mid \varepsilon$$

4. $S \to 0S1 \to 01$ and $S \to SS \to S \to 0S1 \to 01$ are two distinct derivations of 01.

Extra Credit: $S \to 0S1S \mid \varepsilon$ generates the language L of all binary strings such that for each $w \in L$,

- w contains the same number of 0's as 1's, **and**,
- each prefix of w contains at least as many 0's as 1's.