

CS3133

HW#6

DUE: Friday, September 30

(18 points) (Kozen, problem 76) Let $L = a^*b^*c^* - \{a^ib^ic^i \mid i \geq 0\}$. That is, L contains all strings of a 's followed by b 's followed by c 's which do **not** have exactly the same numbers of a 's **and** b 's **and** c 's. For example, $aabbac \notin L$, $aaabbbccc \notin L$, $\varepsilon \notin L$ but $aabb \in L$ and $aaabbbcc \in L$.

a Prove that L is a CFL by giving a CFG G such that $L = L(G)$.

b Is your choice of G ambiguous?

c Prove that L is a CFL by giving a PDA M such that $L = L(M)$.

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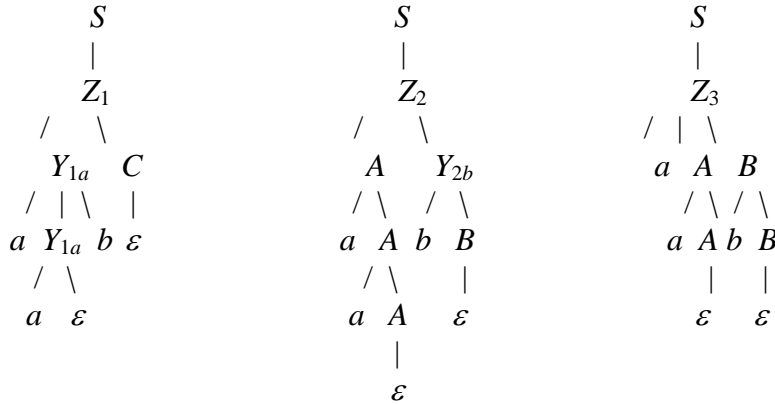
Solutions for HW#6

1. **a** We use A (respectively B and C) to generate a string of a 's (respectively b 's and c 's). The trick will be to cover the three cases:

- Z_1 number of a 's is different than the number of b 's
 - Y_{1a} more a 's than b 's
 - Y_{1b} more b 's than a 's
- Z_2 number of b 's is different than the number of c 's
 - Y_{2b} more b 's than c 's
 - Y_{2c} more c 's than b 's
- Z_3 number of a 's is different than the number of c 's

$$\begin{aligned}
 S &\rightarrow Z_1 \mid Z_2 \mid Z_3 \\
 Z_1 &\rightarrow Y_{1a}C \mid Y_{1b}C \\
 Y_{1a} &\rightarrow aY_{1a}b \mid aA \\
 Y_{1b} &\rightarrow aY_{1b}b \mid bB \\
 Z_2 &\rightarrow AY_{2b} \mid AY_{2c} \\
 Y_{2b} &\rightarrow bY_{2b}c \mid bB \\
 Y_{2c} &\rightarrow bY_{2c}c \mid cC \\
 Z_3 &\rightarrow aZ_3c \mid aAB \mid BcC \\
 A &\rightarrow aA \mid \varepsilon \\
 B &\rightarrow bB \mid \varepsilon \\
 C &\rightarrow cC \mid \varepsilon
 \end{aligned}$$

b The grammar above is ambiguous. There are three parse trees of aab :



c *M* will first "guess" which of the six cases hold:

	<u>state</u>
• more <i>a</i> 's than <i>b</i> 's	$q_{a>b}$
• fewer <i>a</i> 's than <i>b</i> 's	$q_{b>a}$
• more <i>b</i> 's than <i>c</i> 's	$q_{b>c}$
• fewer <i>b</i> 's than <i>c</i> 's	$q_{c>b}$
• more <i>a</i> 's than <i>c</i> 's	$q_{a>c}$
• fewer <i>a</i> 's than <i>c</i> 's	$q_{c>a}$

$$\delta(s, \varepsilon, \perp) = \{(q_{a>b}, \perp), (q_{b>a}, \perp), (q_{b>c}, \perp), (q_{c>b}, \perp), (q_{a>c}, \perp), (q_{c>a}, \perp)\}$$

We list the details of how to proceed from the first case, and the other five are defined in a similar way.

$q_{a>b}$ As *a*'s are read, they are stacked.

$q_{a>b}^*$ As *b*'s are read, *as* are popped from the top of the stack.

$q_{a>b}^{**}$ A string of *c*'s is read, and the stack is not touched.

If any *a*'s are left in the stack, final state q^* is entered.

$$\delta(q_{a>b}, a, \perp) = \{(q_{a>b}, a \perp)\}$$

$$\delta(q_{a>b}, a, a) = \{(q_{a>b}, aa)\}$$

$$\delta(q_{a>b}, b, a) = \{(q_{a>b}^*, \varepsilon)\}$$

$$\delta(q_{a>b}^*, b, a) = \{(q_{a>b}^*, \varepsilon)\}$$

$$\delta(q_{a>b}^*, c, a) = \{(q_{a>b}^{**}, \varepsilon), (q^*, \varepsilon)\}$$

$$\delta(q_{a>b}^{**}, \varepsilon, a) = \{(q^*, \varepsilon)\}$$