CS3133

HW#6

DUE: Friday, September 30

(18 points) (Kozen, problem 76) Let $L = a^*b^*c^* - \{a^ib^ic^i \mid i \geq 0\}$. That is, L contains all strings of a's followed by b's followed by c's which do **not** have exactly the same numbers of a's **and** b's **and** c's. For example, $aabbac \notin L$, $aaabbbccc \notin L$, $\varepsilon \notin L$ but $aabb \in L$ and $aabbbcc \in L$.

a Prove that L is a CFL by giving a CFG G such that $L = L(G)$.

b Is your choice of G ambiguous?

c Prove that L is a CFL by giving a PDA M such that $L = L(M)$.

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Solutions for HW#6

1. ***a*** We use A (respectively B and C) to generate a string of a's (respectively b's and c's). The trick will be to cover the three cases:

- Z_1 number of a's is different than the number of b's
  - Y_{1a} more a's than b's
  - Y_{1b} more b's than a's
- Z_2 number of b's is different than the number of c's
  - Y_{2b} more b's than c's
  - Y_{2c} more c's than b's
- Z_3 number of a's is different than the number of c's

$$
\begin{aligned}
S &\to Z_1 \mid Z_2 \mid Z_3 \\
Z_1 &\to Y_{1a}C \mid Y_{1b}C \\
Y_{1a} &\to aY_{1a}b \mid aA \\
Y_{1b} &\to aY_{1b}b \mid bB \\
Z_2 &\to AY_{2b} \mid AY_{2c} \\
Y_{2b} &\to bY_{2b}c \mid bB \\
Y_{2c} &\to bY_{2c}c \mid cC \\
Z_3 &\to aZ_3c \mid aAB \mid BcC \\
A &\to aA \mid \varepsilon \\
B &\to bB \mid \varepsilon \\
C &\to cC \mid \varepsilon
\end{aligned}
$$

b The grammar above is ambiguous. There are three parse trees of *aab*:

```
        S                     S                    S
        |                     |                    |
        Z1                    Z2                   Z3
      /    \                /    \              /  |  \
     Y1a    C              A      Y2b          a   A   B
   /  | \   |             / \    /  \             / \ / \
  a  Y1a  b ε            a   A  b    B           a  A b  B
     / \                    / \      |                |   |
    a   ε                  a   A     ε                ε   ε
                               |
                               ε
```

c *M* will first "guess" which of the six cases hold:

	state
• more *a*'s than *b*'s	$q_{a>b}$
• fewer *a*'s than *b*'s	$q_{b>a}$
• more *b*'s than *c*'s	$q_{b>c}$
• fewer *b*'s than *c*'s	$q_{c>b}$
• more *a*'s than *c*'s	$q_{a>c}$
• fewer *a*'s than *c*'s	$q_{c>a}$

$$\delta(s,\varepsilon,\perp)=\{(q_{a>b},\perp),(q_{b>a},\perp),(q_{b>c},\perp),(q_{c>b},\perp),(q_{a>c},\perp),(q_{c>a},\perp)\}$$

We list the details of how to proceed from the first case, and the other five are defined in a similar way.

$q_{a>b}$ As *a*'s are read, they are stacked.

$q^{*}_{a>b}$ As *b*'s are read, *a*s are popped from the top of the stack.

$q^{**}_{a>b}$ A string of *c*'s is read, and the stack is not touched.

If any *a*'s are left in the stack, final state q^{*} is entered.

$$\delta(q_{a>b},a,\perp)=\{(q_{a>b},a\perp)\}$$

$$\delta(q_{a>b},a,a)=\{(q_{a>b},aa)\}$$

$$\delta(q_{a>b},b,a)=\{(q^{*}_{a>b},\varepsilon)\}$$

$$\delta(q^{*}_{a>b},b,a)=\{(q^{*}_{a>b},\varepsilon)\}$$

$$\delta(q^{*}_{a>b},c,a)=\{(q^{**}_{a>b},\varepsilon),(q^{*},\varepsilon)\}$$

$$\delta(q^{**}_{a>b},\varepsilon,a)=\{(q^{*},\varepsilon)\}$$