CS3133
HW#5 SOLUTIONS

1. G is ambiguous because there are two distinct parse trees (and two distinct leftmost derivations) of **if** c_1 **then if** c_1 **then** a_1 **else** a_1. The distinct leftmost derivations are

$$S \Rightarrow \text{if } C \text{ then } B \Rightarrow \text{if } c_1 \text{ then } B \Rightarrow \text{if } c_1 \text{ then if } C \text{ then } B \text{ else } S \Rightarrow \text{if } c_1 \text{ then if } c_1 \text{ then } B \text{ else } S$$
$$\Rightarrow \text{if } c_1 \text{ then if } c_1 \text{ then } A \text{ else } S \Rightarrow \text{if } c_1 \text{ then if } c_1 \text{ then } a_1 \text{ else } S \Rightarrow \text{if } c_1 \text{ then if } c_1 \text{ then } a_1 \text{ else } B$$
$$\Rightarrow \text{if } c_1 \text{ then if } c_1 \text{ then } a_1 \text{ else } A \Rightarrow \text{if } c_1 \text{ then if } c_1 \text{ then } a_1 \text{ else } a_1$$

and

$$S \Rightarrow B \Rightarrow \text{if } C \text{ then } B \text{ else } S \Rightarrow \text{if } c_1 \text{ then } B \text{ else } S \Rightarrow \text{if } c_1 \text{ then if } C \text{ then } B \text{ else } S$$
$$\Rightarrow \text{if } c_1 \text{ then if } c_1 \text{ then } B \text{ else } S \Rightarrow \text{if } c_1 \text{ then if } c_1 \text{ then } A \text{ else } S \Rightarrow \text{if } c_1 \text{ then if } c_1 \text{ then } a_1 \text{ else } S$$
$$\Rightarrow \text{if } c_1 \text{ then if } c_1 \text{ then } a_1 \text{ else } B \Rightarrow \text{if } c_1 \text{ then if } c_1 \text{ then } a_1 \text{ else } A \Rightarrow \text{if } c_1 \text{ then if } c_1 \text{ then } a_1 \text{ else } a_1$$

2. The stack has only 0's or only 1's on top of Z_0. When in q_0 and there are 1's in the stack and a 1 is read, then we push the new 1 onto the stack; when a 0 is read we pop the 1 from the stack. We do the opposite when there are 0's in the stack. When the stack contains only Z_0, we allow an e-transition to q_1, the final state.

$P = (\{q_0, q_1\}, \{0,1\}, \{Z_0, 0, 1\}, d, q_0, Z_0, \{q_1\})$ where

$$d(q_0, e, Z_0) = \{(q_1, e)\}$$
$$d(q_0, 0, Z_0) = \{(q_0, 0Z_0)\}$$
$$d(q_0, 0, 0) = \{(q_0, 00)\}$$
$$d(q_0, 0, 1) = \{(q_0, e)\}$$
$$d(q_0, 1, Z_0) = \{(q_0, 1Z_0)\}$$
$$d(q_0, 1, 1) = \{(q_0, 11)\}$$
$$d(q_0, 1, 0) = \{(q_0, e)\}$$

We note that P accepts L by final state **and** by empty stack.

$$(q_0, 110001, Z_0) \vdash (q_0, 10001, 1Z_0) \vdash (q_0, 0001, 11Z_0) \vdash (q_0, 001, 1Z_0) \vdash (q_0, 01, Z_0) \vdash$$
$$(q_0, 1, 0Z_0) \vdash (q_0, e, Z_0) \vdash (q_1, e, e)$$