1. **Proof:** (By contradiction) Assume $L$ (over $\{a,b\}^*$) is regular. Then $\overline{L}$ must be regular. Clearly $L_1 = a^* b^*$ is regular. Thus $\overline{L} \cap L_1$ must also be regular. But $\overline{L} \cap L_1 = \{a'b'|i \in N\}$, which we have shown is not regular. Hence $L$ could not be regular.

2. a) $L_1 = \{a,b\}^*$ is regular, since it is generated by $S \to aS|bS|\lambda$. $L_2 = \{a'b'|i \in N\}$ is not regular. $L_1 \cup L_2 = L_4$, which is regular.
   b) The regular grammar $S \to aA$ generates $L_1 = \emptyset$. $L_2 = \{a'b'|i \in N\}$. $L_1 \cup L_2 = L_2$ which is not regular.
   c) The regular grammar $S \to aA$ generates $L_1 = \emptyset$. $L_2 = \{a'b'|i \in N\}$. $L_1 \cap L_2 = \emptyset$ which is not regular.
   d) $L_1 = \{a'b^i|j \in N \land i \neq j\}$ is not regular. $L_2 = \{a'b^|i \in N\}$ is not regular. $L_1 \cup L_2 = a^* b^*$, which is regular.
   e) It is shown in Example 7.6.1 (and in class) that $L = \{z \in \{a,b\}^{|\text{length}(z)\text{is a perfect square}}\}$ is not regular. But $\{a,b\} \subseteq L$, so $L^* = \{a,b\}^*$, which is regular, since it is generated by $S \to aS|bS|\lambda$.

3. a) $M = (Q = \{q_0,q_1,q_2\}, \Sigma = \{a,b\}, \Gamma = \{A\}, \delta, q_0, F = \{q_0,q_2\})$ accepts $\{a^n b^{2n}|n \in N\}$, where
   \[\delta(q_0,a,\lambda) = [[q_1,A]]\] \[\delta(q_1,\lambda,\lambda) = [[q_0,A]]\] (for every $a$ read, push two $A$’s onto the stack),
   \[\delta(q_0,\lambda,\lambda) = [[q_2,\lambda]]\] (guess that you’re done reading $as$), \[\delta(q_2,b,A) = [[q_2,\lambda]]\].
   b) $S \to aSbb|\lambda$