CS3133
HW #5 SOLUTIONS

1. **<u>PROOF</u>**: (By contradiction) Assume L (over $\{a,b\}^*$) is regular. Then $\overline{L}$ must be regular. Clearly $L_1 = a^*b^*$ is regular. Thus $\overline{L} \cap L_1$ must also be regular. But $\overline{L} \cap L_1 = \{a^i b^i \mid i \in N\}$, which we have shown is not regular. Hence L could not be regular.

2. *a*) $L_1 = \{a,b\}^*$ is regular, since it is generated by $S \to aS \mid bS \mid \lambda$. $L_2 = \{a^i b^i \mid i \in \mathbb{N}\}$ is not regular. $L_1 \cup L_2 = L_1$, which is regular.
b) The regular grammar $S \to aA$ generates $L_1 = \varnothing$. $L_2 = \{a^i b^i \mid i \in \mathbb{N}\}$. $L_1 \cup L_2 = L_2$ which is not regular.
c) The regular grammar $S \to aA$ generates $L_1 = \varnothing$. $L_2 = \{a^i b^i \mid i \in \mathbb{N}\}$. $L_1 \cap L_2 = \varnothing$ which is not regular.
d) $L_1 = \{a^i b^j \mid i, j \in \mathbb{N} \wedge i \neq j\}$ is not regular. $L_2 = \{a^i b^i \mid i \in \mathbb{N}\}$ is not regular. $L_1 \cup L_2 = a^*b^*$, which is regular.
e) It is shown in **Example 7.6.1** (and in class) that $L = \{z \in \{a,b\}^* \mid length(z) \text{ is a perfect square}\}$ is not regular. But $\{a,b\} \subseteq L$, so $L^* = \{a,b\}^*$, which is regular, since it is generated by $S \to aS \mid bS \mid \lambda$.

3. *a*) $M = (Q = \{q_0, q_1, q_2\}, \Sigma = \{a,b\}, \Gamma = \{A\}, \delta, q_0, F = \{q_0, q_2\})$ accepts $\{a^n b^{2n} \mid n \in \mathbb{N}\}$, where $\delta(q_0, a, \lambda) = \{[q_1, A]\}$, $\delta(q_1, \lambda, \lambda) = \{[q_0, A]\}$ (for every *a* read, push two *A*'s onto the stack), $\delta(q_0, \lambda, \lambda) = \{[q_2, \lambda]\}$ (guess that you're done reading *a*s), $\delta(q_2, b, A) = \{[q_2, \lambda]\}$.
b) $S \to aSbb \mid \lambda$