CS3133

HW#5

DUE: Friday, September 24

1. (8 points) Give a context-free grammar for the set of binary strings which are not palindromes.

2. (14 points) Prove or give a counterexample to each of the following CONJECTURES.

CONJECTURE 1: For any CFGs $G_1 = (N_1, \Sigma, P_1, S_1)$ and $G_2 = (N_2, \Sigma, P_2, S_2)$ with $N_1 \cap N_2 = \varnothing$ and $L(G_1) \cap L(G_2) = \varnothing$, if $G^* = (N_1 \cup N_2, \Sigma, P_1 \cup P_2 \cup \{S_1 \to S_2\}, S_1)$, then $L(G^*) \subseteq L(G_1) \cup L(G_2)$.

CONJECTURE 2: For any CFGs $G_1 = (N_1, \Sigma, P_1, S_1)$ and $G_2 = (N_2, \Sigma, P_2, S_2)$ with $N_1 \cap N_2 = \varnothing$ and $L(G_1) \cap L(G_2) = \varnothing$, if $G^* = (N_1 \cup N_2, \Sigma, P_1 \cup P_2 \cup \{S_1 \to S_2\}, S_1)$, then $L(G^*) = L(G_1) \cup L(G_2)$.

CS3133

Solutions to HW#5

1. If a binary string is not a palindrome, then either

- it is $0w0$ or $1w1$ where w is not a palindrome, or,
- it is $0w1$ or $1w0$ where w is a binary string (generated by A below).

$$S \to 0S0|1S1|0A1|1A0$$
$$A \to 0A|1A|\varepsilon$$

2. Both CONJECTURE s are shown to be false by the same counterexample. Let G_1 have productions $S_1 \to 0S_1|\varepsilon$ and G_2 have productions $S_2 \to 1S_2|1$. Then indeed $L(G_1) = 0^*$ and $L(G_2) = 11^*$, and $N_1 \cap N_2 = \varnothing$ and $L(G_1) \cap L(G_2) = \varnothing$. But $01 \in L(G^*)$ although $01 \notin L(G_1) \cup L(G_2)$. So $L(G^*) \neq L(G_1) \cup L(G_2)$.