CS3133

HW#5

DUE: Monday, September 26

1. (8 points) Show that the set of all binary strings with more 1's than 0's is a context free language.

2. (10 points) Prove or give a counterexample to each of the following:
CONJECTURE *A*: For any context free languages L_0 and L_1, language $L_0 \cup L_1$ must be context free.
CONJECTURE *B*: For any context free languages L_0 and L_1, language $L_0 L_1$ must be context free.

3. (12 points) Are either of the following languages context free? Justify your responses.
a $\left\{a^i b^j c^j d^i \mid i, j \geq 0\right\}$.
b $\left\{a^i b^i c^i d^i \mid i \geq 0\right\}$.

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Solutions for HW#5

1. We prove that this language is context free by designing a PDA which accepts with final state. Virtually all computations take place in start state *s*. In state *s*, if the stack contains 0's (respectively, 1's), then the stack contains a 0 (resp., 1) for the excess of 0's (resp., 1's) which have been read. The language is accepted by

$M=\left(\{s,t\},\{0,1\},\{0,\perp\},\delta,s,\perp,\{t\}\right)$, where

$\delta(s,0,\perp)=\{(s,0\perp)\}$, $\delta(s,1,\perp)=\{(s,1\perp)\}$

$\delta(s,0,0)=\{(s,00)\}$, $\delta(s,1,1)=\{(s,11)\}$

$\delta(s,0,1)=\{(s,\varepsilon)\}$, $\delta(s,1,0)=\{(s,\varepsilon)\}$

$\delta(s,\varepsilon,1)=\{(t,\varepsilon)\}$ More 1's have been read than 0's, go to final state

2. Both conjectures are true. If L_0 and L_1 are context free, then there are context free grammars $G_0=(N_0,\Sigma,P_0,S_0)$ and $G_1=(N_1,\Sigma,P_1,S_1)$ such hat $L(G_0)=L_0$, $L(G_1)=L_1$ and $N_0\cap N_1=\varnothing$. Grammar $\left(N_0\cup N_1\cup\{S\},\Sigma,P_0\cup P_1\cup\{S\to S_0\mid S_1\},S\right)$ generates $L_0\cup L_1$, and grammar $\left(N_0\cup N_1\cup\{S\},\Sigma,P_0\cup P_1\cup\{S\to S_0S_1\},S\right)$ generates L_0L_1.

3. ***a*** It is generated by

$$S\to aSd\mid A$$
$$A\to bAc\mid\varepsilon$$

b $\{a^ib^ic^id^i\mid i\geq 0\}$ is not context free. To see this, we assume that it is context free, and we let *n* be the constant guaranteed by the Pumping Lemma for CFLs, and let $z=a^kb^kc^kd^k$. For any decomposition of *z* as *uvwxy* with $|vwx|\leq k$ and $|vx|\geq 1$. Thus, *vx* is not empty and it can not contain *a*'s, *b*'s, *c*'s **and** *d*'s. So uv^2wx^2y can not contain the same number of *a*'s, *b*'s, *c*'s and *d*'s, so it can not belong to $\{a^ib^ic^id^i\mid i\geq 0\}$. But the Pumping Lemma asserts that it must belong. This contradiction shows that $\{a^ib^ic^id^i\mid i\geq 0\}$ can not be context free.