HW#4 SOLUTIONS

(a) Essentially we use A to generate $\{0^n1^n \mid n \geq 0\}$ and B to generate $\{1^k \mid k > 0\}$, a nonempty string of 1s.

$S \to AB$

$A \to 0A1 \mid e$

$B \to 1B \mid 1$

(b) Assume that $w \in L$. Then $w = 0^n1^n1^k$, $k > 0, n \geq 0$. We will prove that $S \Rightarrow AB \overset{*}{\Rightarrow} w$ by induction on n ($A \overset{*}{\Rightarrow} 0^n1^n$) and then induction on k ($B \overset{*}{\Rightarrow} 0^k$).

If n=0, then $A \Rightarrow e$ and we are done. Assuming $A \overset{*}{\Rightarrow} 0^n1^n$, then production $A \to 0A1$ yields $A \Rightarrow 0A1 \overset{*}{\Rightarrow} 0\,0^n1^n1 = 0^{n+1}1^{n+1}$, so A derives arbitrarily long, nonempty strings of the form 0^n1^n.

If k=1, then $B \Rightarrow 1$ and we are done. Assuming $B \overset{*}{\Rightarrow} 1^k$, then production $B \to 1B$ yields $B \Rightarrow 1B \overset{*}{\Rightarrow} 11^k = 1^{k+1}$, so B derives arbitrarily long, nonempty strings of 1s. Thus $L(G) \subseteq L$.

Assume that $w \in L(G)$. Consider a rightmost derivation of w. All derivations in G start with $S \Rightarrow AB$. If there are 0 applications of production $B \to 1B$, then $S \Rightarrow AB \Rightarrow A1$.

(c) Assume L is regular. Then let n be the constant specified by the PUMPING LEMMA FOR REGULAR LANGUAGES, and let $w = 0^n1^{n+1} \in L$. Then w can be written w=xyz where $|xy| \leq n$ (xy spans only 0s) and $y \neq e$, so y must be a nonempty string of 0s. It follows that $xy^3z \in L$, although xy^3z has more 0s than 1s. This contradiction shows that our assumption that L is regular must be false.