CS3133

HW#4

DUE: Monday, September 19

1. (6 points) For the CFG G

$$S \to S + T \mid T$$
$$T \to T * F \mid F$$
$$F \to (S) \mid x$$

and for each of the following strings, give as many parse trees as possible for the string to show that it belongs to $L(G)$.

a x
b $x + x + x$
c $x * ((x)) * x$

2. (8 points) Show that $\left\{(01)^i 0^j \mid i > j \geq 0\right\}$ is not regular.

3. (16 points) ***a*** Is the language $\left\{a^i b^j c^k \mid 0 \leq i, 0 \leq j \leq k\right\}$ regular? Justify your answer.

b Prove that $\left\{a^i b^j c^k \mid 0 \leq i, 0 \leq j \leq k\right\}$ is a CFL by providing a CFG without ε–productions and unit productions to generate $\left\{a^i b^j c^k \mid 0 \leq i, 0 \leq j \leq k\right\} - \{\varepsilon\}$.

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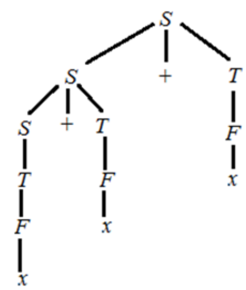
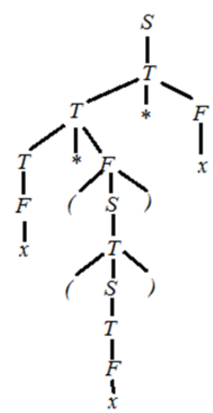
Solutions to HW#4

1. ***a***

b



c



2. Assume $\left\{(01)^i 0^j \mid i > j \geq 0\right\}$ is regular. Let *n* be the number guaranteed by the Pumping Lemma. Choose $(01)^{n+1} 0^n \in \left\{(01)^i 0^j \mid i > j \geq 0\right\}$. Any decomposition of $(01)^{n+1} 0$ into *uvw* with $|uv| \leq n$ and $|v| \geq 1$ has *v* be a nonempty part of $(01)^{n+1}$. If *v*=0 or *v*=1, then $uv^0 w \notin \left\{(01)^i 0^j \mid i > j \geq 0\right\}$. Likewise, if $|v| > 1$ then $uv^2 w \notin \left\{(01)^i 0^j \mid i > j \geq 0\right\}$. So by contradiction $\left\{(01)^i 0^j \mid i > j \geq 0\right\}$ can not be regular.

3. ***a*** Assume $\left\{a^i b^j c^k \mid 0 \leq i, 0 \leq j \leq k\right\}$ is regular, and let *n* be the number guaranteed by the Pumping Lemma. Choose $b^{n+1} c^{n+1} \in \left\{a^i b^j c^k \mid 0 \leq i, 0 \leq j \leq k\right\}$. Any decomposition of $b^{n+1} c^{n+1}$

into *uvw* with $|uv| \le n$ and $|v| \ge 1$ has *v* being a nonempty sequence of *b*s. “Pumping up” the *b*s, the Pumping Lemma assures us that $uv^2w \in \{a^i b^j c^k \mid 0 \le i, 0 \le j \le k\}$. But uv^2w has more *b*s than *c*s. This contradiction yields that $\{a^i b^j c^k \mid 0 \le i, 0 \le j \le k\}$ can not be regular.

b We first provide a CFG to generate $\{a^i b^j c^k \mid 0 \le i, 0 \le j \le k\}$.

$$S \to AB$$
$$A \to aA \mid \varepsilon$$
$$B \to CD$$
$$C \to bCc \mid \varepsilon$$
$$D \to cD \mid \varepsilon$$

We then remove ε–productions by first computing NULLABLE=*N*={*S*, *A*, *B*, *C*, *D*}. Then we contruct the following CFG without ε–productions to generate $\{a^i b^j c^k \mid 0 \le i, 0 \le j \le k\} - \{\varepsilon\}$.

$$S \to AB \mid A \mid B$$
$$A \to aA \mid a$$
$$B \to CD \mid C \mid D$$
$$C \to bCc \mid bc$$
$$D \to cD \mid c$$

We then remove unit productions to the following CFG without ε–productions or unit productions to generate $\{a^i b^j c^k \mid 0 \le i, 0 \le j \le k\} - \{\varepsilon\}$.

$$S \to AB \mid aA \mid a \mid CD \mid bCc \mid bc \mid cD \mid c$$
$$A \to aA \mid a$$
$$B \to CD \mid bCc \mid bc \mid cD \mid c$$
$$C \to bCc \mid bc$$
$$D \to cD \mid c$$