CS3133

HW#4

DUE: Friday, September 17

1. (6 points) Prove or give a counterexample to the following CONJECTURES.

CONJECTURE 1: For any NFA $N = (Q, \Sigma, \Delta, S, F)$ and any $q \in Q$, the language $L_{N,\to q} = \left\{ w \in \Sigma^* \mid (\exists s \in S)\, q \in \hat{\Delta}(s, w) \right\}$ must be regular.

CONJECTURE 2: For any NFA $N = (Q, \Sigma, \Delta, S, F)$ and any $q \in Q$, the language $L_{N,q\to} = \left\{ w \in \Sigma^* \mid (\exists q^* \in F)\, q^* \in \hat{\Delta}(q, w) \right\}$ must be regular.

2. (14 points) The operations $\odot$ and $\otimes$ over languages are defined by

$$L_0 \odot L_1 = \left\{ w \mid (\exists x \in L_1)\, wx \in L_0 \right\}$$

$$L_0 \otimes L_1 = \left\{ wx \mid (\exists x \in L_1)\, w \in L_0 \right\}.$$

Prove or give a counterexample to the following CONJECTURES. Hint: You may want to invoke your answers to Problem 1 above.

CONJECTURE 1: The set of regular languages is closed under the operation $\odot$.

CONJECTURE 2: The set of regular languages is closed under the operation $\otimes$.

3. (10 points) ***a*** Is $\left\{ a^i b^i \mid i \geq 1 \right\} \cup \left\{ a^i b^j \mid i, j \geq 0 \right\}$ regular? Justify your answer.

b Is $\left\{ a^i b^i c^j \mid i \geq 1 \wedge j \geq 0 \right\} \cup \left\{ a^i b^j \mid i, j \geq 0 \right\}$ regular? Justify your answer.

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Solutions to HW#4

1. ***a*** The CONJECTURE is true because $L_{N,\to q}$ is accepted by the NFA $N=\left(Q,\Sigma,\Delta,S,\{q\}\right)$.
b The CONJECTURE is true because $L_{N,q\to}$ is accepted by the NFA $N=\left(Q,\Sigma,\Delta,\{q\},F\right)$.

2. Both CONJECTUREs are true.
CONJECTURE 1: If L_0 and L_1 are regular, there must exist DFAs $M_0=\left(Q_0,\Sigma,\delta_0,s_0,F_0\right)$ and $M_1=\left(Q_1,\Sigma,\delta_1,s_1,F_1\right)$ such that $L_0=L\left(M_0\right)$ and $L_1=L\left(M_1\right)$. From Problem 1 we know that $L_{M_0,q\to}$ is regular for each $q\in Q_0$. And because regular languages are closed under intersection, then $L_{M_0,q\to}\cap L_1$ must be regular. Because it is decidable whether a regular language is empty, we can decide for each $q\in Q_0$ whether $L_{M_0,q\to}\cap L_1$ is empty. So $L_0\odot L_1$ is accepted by $M^*=\left(Q_0,\Sigma,\delta_0,s_0,F^*\right)$ where $F^*=\left\{q\in Q_0 \mid L_{M_0,q\to}\cap L_1\neq\varnothing\right\}$. That is, we accept $w\in\Sigma^*$ if there exists a string $x\in\Sigma^*$ such that $\widehat{\delta}_1\left(s_1,x\right)\in F_1$ and $\widehat{\delta}_0\left(\widehat{\delta}_0\left(s_0,w\right),x\right)\in F_0$.
CONJECTURE 2: $L_0\otimes L_1=L_0L_1$ and we know that the regular languages are closed under concatenation. So they are closed under $\otimes$.

3. ***a*** $\left\{a^ib^i \mid i\geq 1\right\}\cup\left\{a^ib^j \mid i,j\geq 0\right\}=\left\{a^ib^j \mid i,j\geq 0\right\}$, and it is regular because it is accepted by the DFA $M=\left(\left\{q_a,q_b,q_{blackhole}\right\},\{a,b\},\delta,q_a,\left\{q_a,q_b\right\}\right)$ where

δ	a	b
q_a	q_a	q_b
q_b	$q_{blackhole}$	q_b
$q_{blackhole}$	$q_{blackhole}$	$q_{blackhole}$

.

b $\left\{a^ib^ic^j \mid i\geq 1\wedge j\geq 0\right\}\cup\left\{a^ib^j \mid i,j\geq 0\right\}$ is not regular. Let k be as specified by the Pumping Lemma for Regular Languages. We note that $a^kb^kc\in\left\{a^ib^ic^j \mid i\geq 1\wedge j\geq 0\right\}\cup\left\{a^ib^j \mid i,j\geq 0\right\}$. If the language were regular, then a^kb^kc could be written as uvw with $1\leq|v|\leq k$ and $|uv|\leq k$ and $\left\{uv^lw \mid l\geq 0\right\}\subseteq\left\{a^ib^ic^j \mid i\geq 1\wedge j\geq 0\right\}\cup\left\{b^ic^j \mid i,j\geq 0\right\}$. But v must be a nonempty string of a's. So uv^2w has more a's than b's, and it can't belong to $\left\{a^ib^ic^j \mid i\geq 1\wedge j\geq 0\right\}$. And because it doesn't contain a c it can't belong to $\left\{a^ib^j \mid i,j\geq 0\right\}$. So, by contradiction, $\left\{a^ib^ic^j \mid i\geq 1\wedge j\geq 0\right\}\cup\left\{a^ib^j \mid i,j\geq 0\right\}$ is not regular.