CS3133

HW#4

DUE: Monday, September 24

1. (8 points) We say that a CFG is in *Antoniya Normal Form* (ANF) if every production is of the form $A \to aB$ or $A \to a$, with $A, B \in N$ and $a \in \Sigma$. Prove that every grammar in ANF generates a regular language. That is, if there is a CFG *G* in ANF, then *L*(*G*) is regular. (Hint: What aspect of a machine corresponds to the nonterminals?)

2. (10 points) We call CFG $G = (N, \Sigma, P, S)$ *parasite-free* if for all $A \in N$ there exists a string $w \in \Sigma^*$ such that $A \xrightarrow[G]{*} w$. Prove or give a counterexample to the following.

   CONJECTURE: For any context free language *L* there exists a parasite-free CFG grammar *G* such that $L = L(G)$.

3. Give a context free grammar for each of the following languages.

a (1 point) The set of all palindromes over {0,1}. Note that a palindrome can be of even or odd length.

b (2 points) $\{0^m 1^n \mid m \neq n\}$

c (2 points) $\{a^m b^n c^n d^{2m} \mid m, n \geq 0\}$

c (2 points) $\{a^m b^n c^k \mid (m, n \geq 0) \wedge (m = n \vee n = k)\}$

4. (8 points) Give a grammar equivalent to

$$S \to 00A0 \mid B \mid 1C \mid 0D$$
$$A \to \varepsilon \mid 00 \mid 10B$$
$$B \to A \mid 1D$$
$$D \to 010 \mid A$$

which does not have any unit productions or $\varepsilon -$ productions.

CS3133

Solutions for HW#4

1. Given CFG $G=(N,\Sigma,P,S)$ we construct NFA $\left(N\cup\{q_f\},\Sigma,\Delta,\{S\},\{q_f\}\right)$. A state $A\in N$ corresponds to an *A* in a sentential form $S\xrightarrow{*}\alpha A, \alpha\in\Sigma^*$. After having consumed α, we need to "get rid of" *A* by tracing a path through the NFA corresponding to applying a sequence of productions of *P*. For each $A\in N, a\in\Sigma$ we have

$$\Delta(A,a)=\begin{cases}\{B\,|\,(A\to aB)\in P\}, \text{if } (A\to a)\notin P\\ \{B\,|\,(A\to aB)\in P\}\cup\{q_f\}, \text{if } (A\to a)\in P\end{cases},$$

and for each $a\in\Sigma$, $\Delta(q_f,a)=\varnothing$

2. *NONPARASITE* is the set of nonterminals which derive strings in Σ^*. Removing all parasites from a grammar does not change the language it generates.

Alg: $NonParasite\leftarrow\{A\in N\,|\,(A\to w)\in P \text{ where } w\in\Sigma^*\}$

repeat

$(\forall A\in N-NonParasite)$

if $A\to w\in P\wedge w\in(\Sigma\cup NonParasite)^*$ **then** $NonParasite\leftarrow NonParasite\cup\{A\}$

until no change in *NonParasite*

Remove all rules with a nonterminal from *N*- *NONPARASITE*.

3. ***a***

$S\to 0S0\,|\,1S1\,|\,\varepsilon\,|\,0\,|\,1$

b

$S\to 0S1\,|\,A_{m>n}\,|\,A_{n>m}$

$A_{m>n}\to 0A_{m>n}\,|\,0$

$A_{n>m}\to 1A_{n>m}\,|\,1$

c

$S\to aSdd\,|\,A$

$A\to bAc\,|\,\varepsilon$

d

$S\to A_{m=n}B_c\,|\,B_aA_{n=k}$

$A_{m=n}\to aA_{m=n}b\,|\,\varepsilon$

$A_{n=k}\to bA_{n=k}c\,|\,\varepsilon$

$B_c\to cB_c\,|\,\varepsilon$

$B_a\to aB_a\,|\,\varepsilon$

4. Every symbol in N is nullable. Removing $\varepsilon-$productions yields

$$S \to 00A0 \mid 000 \mid B \mid 1C \mid 1 \mid 0D \mid 0$$

$$A \to 00 \mid 10B \mid 10$$

$$B \to A \mid 1D \mid 1$$

$$D \to 010 \mid A$$

To remove unit productions, we first compute $\textsc{Chain}(S)=\{S,B,A\}$, $\textsc{Chain}(A)=\{A\}$, $\textsc{Chain}(B)=\{B,A\}$ $\textsc{Chain}(D)=\{D,A\}$. Then we remove the unit productions

$$S \to 00A0 \mid 000 \mid 1C \mid 1 \mid 0D \mid 0 \mid 1D \mid 1 \mid 00 \mid 10B \mid 10$$

$$A \to 00 \mid 10B \mid 10$$

$$B \to 1D \mid 1 \mid 00 \mid 10B \mid 10$$

$$D \to 010 \mid 00 \mid 10B \mid 10$$