CS3133  
HW#3 SOLUTIONS

1. a) \((0+1)^* 11(0+1)^*\)
b) Every 1 must either be followed by a 0 or be the last character of the string. 
\((0+10)^* (\varepsilon + 1)\)
or, if you don’t want to use \(\varepsilon\) in the regular expression, \((0+10)^* + (0+10)^* 1\).
c) \((0+1)^* 1(0+1)\)

2. The set of binary strings containing 000.

3. a) 
\[
\begin{align*}
R_{11}^0 &= \varepsilon \\
R_{12}^0 &= 0 \\
R_{13}^0 &= 1 \\
R_{21}^0 &= 0 \\
R_{22}^0 &= \varepsilon \\
R_{23}^0 &= 1 \\
R_{31}^0 &= 1 \\
R_{32}^0 &= 0 \\
R_{33}^0 &= \varepsilon \\
\end{align*}
\]
b) 
\[
\begin{align*}
R_{11}^1 &= \varepsilon \\
R_{12}^1 &= 0 \\
R_{13}^1 &= 1 \\
R_{21}^1 &= 0 \\
R_{22}^1 &= \varepsilon + 00 \\
R_{23}^1 &= 1 + 01 \\
R_{31}^1 &= 1 \\
R_{32}^1 &= 0 + 10 \\
R_{33}^1 &= \varepsilon + 11 \\
\end{align*}
\]
c)  
\[ R_{11}^2 = \epsilon + 0(00)^* 0 = (00)^* \]
\[ R_{12}^2 = 0 + 0(00)^* (\epsilon + 00)^* = 0(00)^* \]
\[ R_{13}^2 = 1 + 0(\epsilon + 00)^* (1 + 01) = 1 + 0(00)^* (\epsilon + 0) 1 = 0^* 1 \]
\[ R_{21}^2 = 0 + (\epsilon + 00)(\epsilon + 00)^* 0 = 0 + (00)^* 0 = (00)^* 0 \]
\[ R_{22}^2 = \epsilon + 00 + (\epsilon + 00)(\epsilon + 00)^* (\epsilon + 00) = (00)^* \]
\[ R_{23}^2 = 1 + 01 + (\epsilon + 00)(\epsilon + 00)^* (1 + 01) = (00)^* (1 + 01) \]
\[ R_{31}^2 = 1 + (0 + 10)(\epsilon + 00)^* 0 = 1 + (0 + 10)(00)^* 0 \]
\[ R_{32}^2 = 0 + 10 + (0 + 10)(\epsilon + 00)^* (\epsilon + 00) = (0 + 10)(00)^* \]
\[ R_{33}^2 = \epsilon + 11 + (0 + 10)(\epsilon + 00)^* (1 + 01) = \epsilon + 11 + (0 + 10)(00)^* (1 + 01) \]

d) Our answer is \( R_{13}^2 \), which is
\[ R_{13}^2 = 0^* 1 + 0^* 1 (\epsilon + 11 + (0 + 10)(00)^* (1 + 01))^* (\epsilon + 11 + (0 + 10)(00)^* (1 + 01)) \]
\[ = 0^* 1 (1 + (0 + 10)(00)^* (1 + 01))^* \]