

**CS3133**  
**HW#3 SOLUTIONS**

1. a)  $(0+1)^* 11(0+1)^*$

b) Every 1 must either be followed by a 0 or be the last character of the string.

$$(0+10)^* (\epsilon + 1)$$

or, if you don't want to use  $\epsilon$  in the regular expression,  $(0+10)^* + (0+10)^* 1$ .

c)  $(0+1)^* 1(0+1)$

2. The set of binary strings containing 000.

3. a)

$$R_{11}^0 = \epsilon$$

$$R_{12}^0 = 0$$

$$R_{13}^0 = 1$$

$$R_{21}^0 = 0$$

$$R_{22}^0 = \epsilon$$

$$R_{23}^0 = 1$$

$$R_{31}^0 = 1$$

$$R_{32}^0 = 0$$

$$R_{33}^0 = \epsilon$$

b)

$$R_{11}^1 = \epsilon$$

$$R_{12}^1 = 0$$

$$R_{13}^1 = 1$$

$$R_{21}^1 = 0$$

$$R_{22}^1 = \epsilon + 00$$

$$R_{23}^1 = 1 + 01$$

$$R_{31}^1 = 1$$

$$R_{32}^1 = 0 + 10$$

$$R_{33}^1 = \epsilon + 11$$

**c)**

$$R_{11}^2 = \mathbf{e} + 0(00)^* 0 = (00)^*$$

$$R_{12}^2 = 0 + 0(00)^* (\mathbf{e} + 00)^* = 0(00)^*$$

$$R_{13}^2 = 1 + 0(\mathbf{e} + 00)^* (1 + 01) = 1 + 0(00)^* (\mathbf{e} + 0)1 = 0^*1$$

$$R_{21}^2 = 0 + (\mathbf{e} + 00)(\mathbf{e} + 00)^* 0 = 0 + (00)^* 0 = (00)^* 0$$

$$R_{22}^2 = \mathbf{e} + 00 + (\mathbf{e} + 00)(\mathbf{e} + 00)^* (\mathbf{e} + 00) = (00)^*$$

$$R_{23}^2 = 1 + 01 + (\mathbf{e} + 00)(\mathbf{e} + 00)^* (1 + 01) = (00)^* (1 + 01)$$

$$R_{31}^2 = 1 + (0 + 10)(\mathbf{e} + 00)^* 0 = 1 + (0 + 10)(00)^* 0$$

$$R_{32}^2 = 0 + 10 + (0 + 10)(\mathbf{e} + 00)^* (\mathbf{e} + 00) = (0 + 10)(00)^*$$

$$R_{33}^2 = \mathbf{e} + 11 + (0 + 10)(\mathbf{e} + 00)^* (1 + 01) = \mathbf{e} + 11 + (0 + 10)(00)^* (1 + 01)$$

**d)** Our answer is  $R_{13}^3$ , which is

$$\begin{aligned} R_{13}^2 &= 0^*1 + 0^*1(\mathbf{e} + 11 + (0 + 10)(00)^* (1 + 01))^* (\mathbf{e} + 11 + (0 + 10)(00)^* (1 + 01)) \\ &= 0^*1(1 + (0 + 10)(00)^* (1 + 01))^* \end{aligned}$$